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A Three-variable Isogeometric Model for Free Vibration Analysis of Triply Periodic Minimal Surface Plates

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Abstract: Triply Periodic Minimal Surface (TPMS) architectures offer an optimal balance of minimal weight and superior mechanical resilience. Consequently, they have emerged as excellent candidates for advanced engineering applications, including aerospace and biomedical structures. To better understand their vibrational behavior, this study presents an effective framework dedicated to the free vibration analysis of TPMS-based plates. The computational model employs isogeometric analysis (IGA) in conjunction with a higher-order plate theory. This approach efficiently reduces the required independent variables to three while automatically satisfying stress-free boundary conditions. The reliability of this numerical approach is first confirmed through benchmark comparisons. Subsequently, the study investigates the vibrational characteristics of plates modeled with Gyroid, Primitive, and I-graph and Wrapped Package-graph (IWP) architectures. A key focus is placed on the impact of architectural variation. Therefore, uniform (Uni.), symmetric (Sym.), and asymmetric (Asym.) porosity distributions along the plate's thickness are systematically analyzed. The computational results indicate a profound sensitivity of the natural frequencies to both the selected porosity distribution functions and the underlying TPMS geometries. Ultimately, these findings demonstrate the critical role of these parameters in optimizing lightweight, high-performance structural designs.

Keywords: Free Vibration Analysis; Lightweight Structures; Triply Periodic Minimal Surface (TPMS); Three-Variable Plate Theory; Isogeometric Analysis (IGA).

1. Introduction

Currently, scientists are actively exploring and developing lightweight materials that exhibit high strength and superior mechanical performance. Among these, TPMS structures are attracting significant scientific interest due to their exceptional mechanical properties and high stiffness-to-weight ratios. These advantages are primarily attributed to their inherent periodicity, smooth curvature, and continuous surface

geometry [1, 2]. Furthermore, the rapid advancement of additive manufacturing technologies in recent years has eliminated traditional fabrication barriers, enabling the precise 3D printing of these complex microstructures and unlocking numerous potential applications in modern engineering fields [3, 4]. Although various TPMS topologies have been proposed and investigated, the Primitive (P), Gyroid (G), and I-graph Wrapped Package (IWP) structures remain

the three most widely utilized and analyzed architectures, owing to their optimal stress dispersion capabilities and unique geometric characteristics [5, 6]. However, analyzing the structural behavior of these TPMS models poses several significant challenges, primarily due to their complicated geometries.

To improve geometric approximation accuracy and maintain higher-order continuity, Hughes et al. [7] introduced IGA. This method bridges the gap between design and analysis by integrating the exact CAD geometry directly into the computational framework, thereby minimizing the geometric approximations often associated with traditional meshing. By directly utilizing Non-Uniform Rational B-Splines (NURBS) basis functions from Computer-Aided Design (CAD) systems, IGA enables the preservation of the smoothness and continuity of the original geometry throughout the computational process. This approach helps minimize errors arising from geometric discretization, while providing higher-order continuity between elements. Although several recent studies have successfully employed traditional finite element models and semi-analytical approaches to evaluate the dynamic responses of complex porous and functionally graded structures [8, 9], traditional FEA basis functions are typically limited to C_0 -continuity across element boundaries. In contrast, NURBS basis functions of degree p offer C_{p-1} -continuity. This superior smoothness significantly mitigates numerical issues, such as the appearance of optical branches in frequency spectra [10]. This mathematical characteristic is highly suitable for solving problems requiring higher-order derivatives, such as the vibration analysis of plate and shell structures, thereby contributing to the enhanced reliability of simulation results [10, 11].

Furthermore, the IGA method is often combined with higher-order shear deformation theories (HSDTs) to model structures with complex material distributions, such as functionally graded

materials, or varying thicknesses [12, 13]. This integration is particularly advantageous because HSDTs account for the parabolic distribution of transverse shear stresses across the thickness, thereby eliminating the need for shear correction factors commonly required in first-order shear deformation theories (FSDTs). Based on this foundation, previous studies have successfully employed the five-variable theory [14, 15] and the four-variable refined plate theory [16, 17] to evaluate the bending and free vibration behaviors of both thin and thick plates. These refined models provide a more comprehensive representation of the kinematic field, enhancing the predictive capability of the analysis for advanced composite structures.

More recently, the three-variable shear deformation plate theory [18, 19] has garnered significant attention from researchers due to the high accuracy of this approach in analyzing functionally graded porous plates. By requiring only three independent variables, this theory substantially reduces the overall computational cost while naturally satisfying the zero-traction shear stress boundary conditions at the top and bottom surfaces of the plate. However, NURBS basis functions possess a non-interpolatory nature. This means their control points generally do not coincide with the physical geometry boundary. Consequently, applying essential boundary conditions in IGA requires special numerical techniques. To address this mathematical characteristic gracefully, robust approaches like the penalty method have been widely adopted. These methods ensure both optimal accuracy and numerical stability without compromising the continuity of the formulation [19-21].

Besides the development of robust computational models, investigating the influence of material distribution on the mechanical responses of advanced structures has become a crucial research direction. For instance, recent studies have highlighted the significant impact of

different porosity distribution patterns on the structural stability and dynamic behaviors of functionally graded plates and shells [22, 23]. Building upon these foundational concepts for advanced architectures like TPMS, introducing variable porosity distributions along specific directions can further optimize the stiffness-to-weight ratio. Several recent studies have actively explored the static behavior, energy absorption, and structural optimization of functionally graded TPMS structures [24, 25]. Furthermore, several pioneering works have focused on the computational modeling and mechanical evaluation of functionally graded TPMS plates [26-30]. However, the complex interaction between different TPMS geometric architectures (Primitive, Gyroid, IWP) and various porosity distribution patterns has not been comprehensively explored, particularly under the framework of a three-variable shear deformation theory combined with the IGA approach.

To fill this research gap, this study introduces an effective numerical framework based on IGA to examine the free vibration behaviors of plate structures composed of TPMS architectures. The computational model implements the three-variable higher-order plate theory, optimizing

computational cost while naturally satisfying shear stress boundary conditions. Through this framework, three distinct TPMS architectures are comprehensively investigated. Additionally, this research incorporates three distinct porosity profiles (Sym., Asym., and Uni.) along the thickness direction to investigate the effect of material distribution on vibrational characteristics. This paper explores the sensitivity of natural vibration frequencies to both geometric configurations and material parameters under various boundary conditions. Through this investigation, we aim to provide profound insights into the dynamic behavior of TPMS plates, highlighting their potential in the design of high-performance, lightweight engineering structures.

2. Theory and formulation

2.1. TPMS plate model

The Primitive, Gyroid, and I-graph Wrapped Package (IWP) surfaces are three widely studied TPMS configurations, each exhibiting unique spatial characteristics such as smooth curvature, periodicity, and non-self-intersecting connectivity across three dimensions. These geometric features are described using implicit mathematical equations, which define the corresponding surface morphologies in Cartesian space as follows [14]

$$\begin{aligned}
 \text{Primitive} \quad & \Phi = \cos(\omega_1 x) + \cos(\omega_2 y) + \cos(\omega_3 z), \\
 \text{Gyroid} \quad & \Phi = \sin(\omega_1 x) \cos(\omega_2 x) + \sin(\omega_2 y) \cos(\omega_3 y) + \sin(\omega_3 z) \cos(\omega_1 z), \\
 \text{IWP} \quad & \Phi = 2 \left[\cos(\omega_1 x) \cos(\omega_2 y) + \cos(\omega_2 y) \cos(\omega_3 z) + \cos(\omega_3 z) \cos(\omega_1 x) \right] \\
 & - \left[\cos(2\omega_1 x) + \cos(2\omega_2 y) + \cos(2\omega_3 z) \right].
 \end{aligned} \tag{1}$$

The parameter ω_ψ represents the angular frequency of the TPMS structure and is defined as

$$\omega_\psi = 2\pi n_\psi / l_\psi, \quad \psi = 1, 2, 3 \tag{2}$$

where n_ψ is the number of unit cells and l_ψ is the corresponding length of the computational domain in each spatial direction. Fig. 1 shows three sheet-based TPMS architectures, including Primitive, Gyroid, and IWP types, all generated with a uniform relative density of $\rho = 0.2$.

In this study, the TPMS plates feature three distinct density distribution patterns along the thickness direction: Asym., Sym., and Uni. These distribution types are mathematically described by specific functions that define the variation of material properties across the plate's thickness, as presented in the following equations [27]

Eq. (3)

where $R_{\max}$, R_0 and M_R denoting the maximum

relative density, the porosity coefficient, and the mean relative mass, respectively. The power indices n_a and n_s in Asym. and Sym. density distributions significantly influence the shape of the

$$\begin{aligned}
 \text{Asym.} \quad \rho_A(z) &= \rho_s R_{\max} \left(1 - R_0 + R_0 \left(\frac{z}{h} + \frac{1}{2} \right)^{n_a} \right), \\
 \text{Sym.} \quad \rho_S(z) &= \rho_s R_{\max} \left(1 - R_0 + R_0 \left(1 - \cos \left(\frac{\pi z}{h} \right) \right)^{n_s} \right), \\
 \text{Uni.} \quad \rho_U(z) &= \rho_s R_{\max} \left(1 - R_0 + R_0 \left(\frac{M_R - R_{\max}(1 - R_0)}{R_{\max} R_0} \right) \right), \\
 E &= E_e \cdot E_s, \quad G = G_e \cdot G_s, \quad K = K_e \cdot K_s, \quad \nu = \frac{3K - E}{6K}.
 \end{aligned} \tag{3}$$

where E_s, G_s , and K_s are the elastic modulus, shear modulus, and bulk modulus of the isotropic material, respectively. Table 1 presents the formulations for determining the effective property of the elastic modulus (E_e), the shear modulus (G_e), and the bulk modulus (K_e) based on the two-phase model, which can be determined as [14].

2.2. Three-variable plate theory

The displacements of the plate structures, formulated within a three-variable plate theory integrating in-plane stretching and through-thickness shear deformation, are mathematically represented as follows [18].

Eq. (5)

herein, u_0 , v_0 and w_0 denote the displacements in the mid-plane. The parameter α represents a correction factor for the transverse shear displacement. Various distribution functions $f(z)$, which describe the through-thickness variation of shear strains and stresses, are detailed in Table 2.

The bending and shear strains are defined by the following expressions

Eq. (6) Eq. (7) Eq. (8)

According to this theory, the resultants can be represented by the following expressions

Eq. (9)

density variation, as described in [27].

The elastic modulus (E), shear modulus (G), and Poisson's ratio (ν) for the TPMS architectures are determined by Eq. (4)

The material matrices are determined by the following expressions

Eq. (10) Eq. (11)

The correction factor α is computed as follows, see [18]

Eq. (12)

The weak form for the free vibration analysis of TPMS plates is given by Eq. (13) in which

Eq. (14) Eq. (15) Eq. (16)

2.3. An approximate model using NURBS-based isogeometric analysis

This study uses the NURBS-based IGA framework to assess the fundamental displacements of the plates, hence enabling the approximation of the basic displacement constituents by the following formulation [18]

Eq. (17)

where $\mathbf{q}_I = [u_{0I} \ v_{0I} \ w_{0I}]^T$ denotes the nodal degrees of freedom (DOFs) associated with the control point I , including displacements along the x , y , and z directions. The matrix function $\mathbf{R}_I(\xi, \eta)$ represents the NURBS shape functions, which form the basis for approximating both the geometry and variables in the isogeometric framework.

Subsequently, the formulation of the strain components using NURBS functions is shown below [18]

Eq. (18)

where

Eq. (19) Eq. (20)

The approximate displacement field $\mathbf{u}$ can now be rewritten as follows [18]

Eq. (21)

where

Eq. (22)

The equation that governs the free vibration analysis of TPMS plates is presented in matrix format as follows [18]

Eq. (23)

In this case, the global stiffness matrix $\mathbf{K}$ as well as the mass matrix $\mathbf{M}$ can be shown by the following equations

Eq. (24) Eq. (25)

and ω is the natural frequency.

It is important to note that the control points of NURBS basis functions do not typically lie directly on the physical boundaries of the plate. Consequently, imposing essential boundary conditions directly into the system equations is challenging. To resolve this issue, the penalty method is adopted in this study. Following the formulation proposed by Nguyen et al. [19], the

boundary constraints are weakly enforced by incorporating a penalty stiffness matrix into the global system. Specifically, the modified global stiffness matrix used in the present model is defined as

Eq. (26)

where p is the penalty parameter, $\mathbf{K}_A^p$ represents the constraint matrix associated with a boundary point A as follows [19]

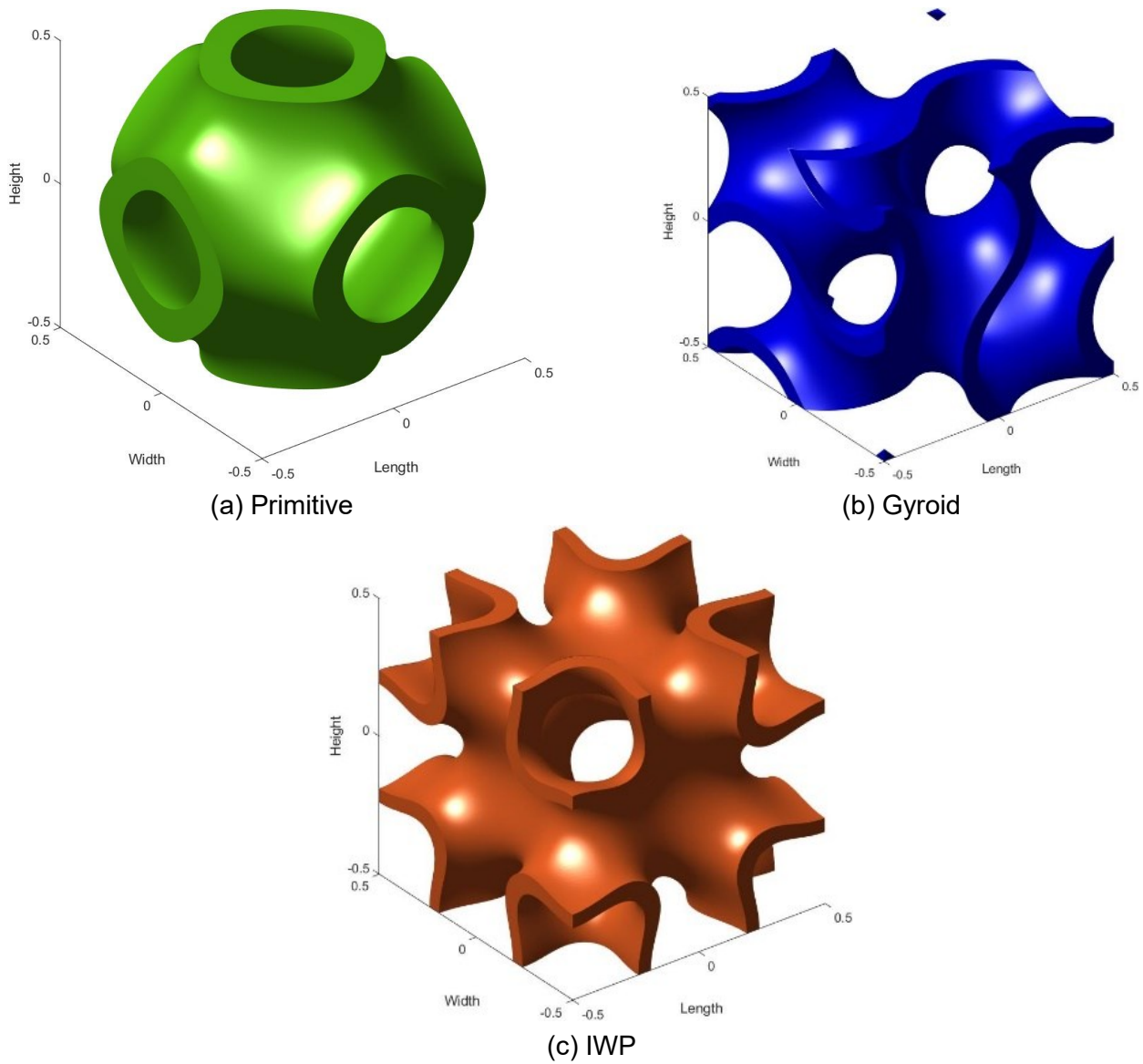
Eq. (27)

While a sufficiently large p is required to satisfy the constraints accurately; an excessively large value may lead to numerical instability [37]. Previous studies have employed varying penalty values depending on the specific nature of the problem; for instance, $p = 10^{10}$ was utilized for the static analysis of plates [19], whereas a penalty parameter of $p = 10^8$ was adopted for transient solutions in cavity problems [38]. Therefore, to ensure both numerical stability and optimal accuracy for the current free vibration analysis of TPMS plates, a specific convergence study of the penalty parameter is systematically conducted in the next section.

Table 1. The effective mechanical properties of TPMS-based unit cells [14].

TPMS Types	Case	Effective property		
		Elastic modulus	Shear modulus	Bulk modulus
Primitive	$\rho \leq 0.25$	$E_e = 0.317\rho^{1.264}$	$G_e = 0.705\rho^{1.189}$	$K_e = 0.487\rho^{1.127}$
	$\rho > 0.25$	$E_e = 1.007\rho^{2.006} - 0.007$	$G_e = 0.953\rho^{1.715} + 0.047$	$K_e = 0.932\rho^{2.380} + 0.068$
Gyroid	$\rho \leq 0.45$	$E_e = 0.596\rho^{1.467}$	$G_e = 0.777\rho^{1.544}$	$K_e = 0.536\rho^{1.240}$
	$\rho > 0.45$	$E_e = 0.962\rho^{2.351} + 0.038$	$G_e = 0.973\rho^{1.982} + 0.027$	$K_e = 0.861\rho^{3.336} + 0.139$
IWP	$\rho \leq 0.35$	$E_e = 0.597\rho^{1.225}$	$G_e = 0.529\rho^{1.287}$	$K_e = 0.439\rho^{1.064}$
	$\rho > 0.35$	$E_e = 0.987\rho^{1.782} + 0.013$	$G_e = 0.960\rho^{2.188} + 0.040$	-

$$\begin{aligned}
 u(x, y, z) &= u_0(x, y) - z \frac{\partial w_0}{\partial x} + \alpha(f(z) - z) \frac{\partial}{\partial x} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right), \\
 v(x, y, z) &= v_0(x, y) - z \frac{\partial w_0}{\partial y} + \alpha(f(z) - z) \frac{\partial}{\partial y} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right), \\
 w(x, y, z) &= w_0(x, y) + \alpha \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right),
 \end{aligned} \tag{5}$$

**Fig. 1.** Representation of three TPMS unit cells**Table 2.** The list of various distribution functions $f(z)$

Case	Source	$f(z)$
Case 1	Nguyen [31]	$f(z) = -8z + \frac{10z^3}{h^2} + \frac{6z^5}{5h^4} + \frac{8z^7}{7h^6}$
Case 2	Reddy [32]	$f(z) = z - \frac{4z^3}{3h^2}$
Case 3	Arya [33]	$f(z) = \sin\left(\frac{\pi z}{h}\right)$
Case 4	Nguyen-Xuan [34]	$f(z) = \arctan\left(\sin\frac{\pi z}{h}\right)$
Case 5	Shimpi [35]	$f(z) = \frac{5z}{4} - \frac{5z^3}{3h^2}$
Case 6	Zenkour [36]	$f(z) = h \sinh\left(\frac{z}{h}\right) - \frac{4z^3}{3h^2} \cosh\left(\frac{1}{2}\right)$

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial y} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{bmatrix} - z \begin{bmatrix} \frac{\partial^2 w_0}{\partial x^2} \\ \frac{\partial^2 w_0}{\partial y^2} \\ 2 \frac{\partial^2 w_0}{\partial x \partial y} \end{bmatrix} + \alpha (f(z) - z) \begin{bmatrix} \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \\ \frac{\partial^2}{\partial y^2} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \\ 2 \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \end{bmatrix} \quad (6)$$

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_1 + z \boldsymbol{\varepsilon}_2 + \alpha (f(z) - z) \boldsymbol{\varepsilon}_3 \quad (7)$$

$$\begin{bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} = \alpha f'(z) \begin{bmatrix} \frac{\partial}{\partial x} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \\ \frac{\partial}{\partial y} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \end{bmatrix} = \alpha f'(z) \boldsymbol{\varepsilon}_s \quad (8)$$

$$\begin{bmatrix} N_{xx} & N_{yy} & N_{xy} \end{bmatrix}^T = \mathbf{A} \boldsymbol{\varepsilon}_1 + \mathbf{B} \boldsymbol{\varepsilon}_2 + \mathbf{G} \boldsymbol{\varepsilon}_3$$

$$\begin{bmatrix} M_{xx} & M_{yy} & M_{xy} \end{bmatrix}^T = \mathbf{B} \boldsymbol{\varepsilon}_1 + \mathbf{D} \boldsymbol{\varepsilon}_2 + \mathbf{H} \boldsymbol{\varepsilon}_3 \quad (9)$$

$$\begin{bmatrix} Q_{xz} & Q_{yz} \end{bmatrix}^T = \mathbf{D}^s \boldsymbol{\varepsilon}_s$$

$$[\mathbf{A}, \mathbf{B}, \mathbf{D}, \mathbf{G}, \mathbf{H}, \mathbf{T}] = \int_{-h/2}^{h/2} [1, z, z^2, \alpha (f(z) - z), \alpha z (f(z) - z), \alpha^2 (f(z) - z)^2] \mathbf{C}_p dz, \quad (10)$$

$$\mathbf{C}_p = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix}, \quad \mathbf{D}^s = \int_{-h/2}^{h/2} \alpha^2 (f'(z))^2 \begin{bmatrix} C_{55} & 0 \\ 0 & C_{44} \end{bmatrix} dz \quad (11)$$

$$\alpha = \frac{z_0 \int_{-h/2}^{h/2} \frac{z E(z)}{1 - \nu^2} dz - \int_{-h/2}^{h/2} \frac{z^2 E(z)}{1 - \nu^2} dz}{\int_{-h/2}^{h/2} f'(z) \frac{E(z)}{2(1 + \nu)} dz}, \quad z_0 = \frac{\int_{-h/2}^{h/2} z E(z) dz}{\int_{-h/2}^{h/2} E(z) dz}. \quad (12)$$

$$\int_{\Omega} \delta \boldsymbol{\varepsilon}_b^T \mathbf{D}^b \boldsymbol{\varepsilon}_b d\Omega + \int_{\Omega} \delta \boldsymbol{\varepsilon}_s^T \mathbf{D}^s \boldsymbol{\varepsilon}_s d\Omega = \int_{\Omega} \delta \hat{\mathbf{u}}^T \ddot{\mathbf{m}} d\Omega, \quad (13)$$

$$\boldsymbol{\varepsilon}_b = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix}, \quad \mathbf{D}^b = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{G} \\ \mathbf{B} & \mathbf{D} & \mathbf{H} \\ \mathbf{G} & \mathbf{H} & \mathbf{T} \end{bmatrix}, \quad \mathbf{m} = \begin{bmatrix} l_1 & l_2 & l_4 \\ l_2 & l_3 & l_5 \\ l_4 & l_5 & l_6 \end{bmatrix}. \quad (14)$$

$$[l_1, l_2, l_3, l_4, l_5, l_6] = \int_{-h/2}^{h/2} \rho [1, z, z^2, \alpha (f(z) - z), \alpha z (f(z) - z), \alpha^2 (f(z) - z)^2] dz, \quad (15)$$

$$\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix}, \quad \mathbf{u}_2 = - \begin{bmatrix} \frac{\partial w_0}{\partial x} \\ \frac{\partial w_0}{\partial y} \\ 0 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} \frac{\partial}{\partial x} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \\ \frac{\partial}{\partial y} \left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \right) \\ \frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} \end{bmatrix}. \quad (16)$$

$$\mathbf{u}^h(\xi, \eta) = \sum_{l=1}^{m \times n} \mathbf{R}_l(\xi, \eta) \mathbf{q}_l, \quad (17)$$

$$[\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3 \quad \varepsilon_s]^T = \sum_{l=1}^{m \times n} [\mathbf{B}_{1l} \quad \mathbf{B}_{2l} \quad \mathbf{B}_{3l} \quad \mathbf{B}_{sl}]^T \mathbf{q}_l = \sum_{l=1}^{m \times n} \bar{\mathbf{B}}_l \mathbf{q}_l, \quad (18)$$

$$\mathbf{B}_{1l} = \begin{bmatrix} \frac{\partial R_l}{\partial x} & 0 & 0 \\ 0 & \frac{\partial R_l}{\partial y} & 0 \\ \frac{\partial R_l}{\partial y} & \frac{\partial R_l}{\partial x} & 0 \end{bmatrix} \quad \mathbf{B}_{2l} = - \begin{bmatrix} 0 & 0 & \frac{\partial^2 R_l}{\partial x^2} \\ 0 & 0 & \frac{\partial^2 R_l}{\partial y^2} \\ 0 & 0 & 2 \frac{\partial^2 R_l}{\partial x \partial y} \end{bmatrix} \quad (19)$$

$$\mathbf{B}_{3l} = \begin{bmatrix} 0 & 0 & \frac{\partial^2}{\partial x^2} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \\ 0 & 0 & \frac{\partial^2}{\partial y^2} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \\ 0 & 0 & 2 \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \end{bmatrix} \quad \mathbf{B}_{sl} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \\ 0 & 0 & \frac{\partial}{\partial y} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \end{bmatrix} \quad (20)$$

$$[\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3]^T = \sum_{l=1}^{m \times n} [\mathbf{N}_{1l} \quad \mathbf{N}_{2l} \quad \mathbf{N}_{3l}]^T \mathbf{q}_l, \quad (21)$$

$$\mathbf{N}_{1l} = \begin{bmatrix} R_l & 0 & 0 \\ 0 & R_l & 0 \\ 0 & 0 & R_l \end{bmatrix} \quad \mathbf{N}_{2l} = - \begin{bmatrix} 0 & 0 & \frac{\partial R_l}{\partial x} \\ 0 & 0 & \frac{\partial R_l}{\partial y} \\ 0 & 0 & 0 \end{bmatrix} \quad \mathbf{N}_{3l} = \begin{bmatrix} 0 & 0 & \frac{\partial}{\partial x} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \\ 0 & 0 & \frac{\partial}{\partial y} \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \\ 0 & 0 & \left(\frac{\partial^2 R_l}{\partial x^2} + \frac{\partial^2 R_l}{\partial y^2} \right) \end{bmatrix} \quad (22)$$

$$(\mathbf{K} - \omega^2 \mathbf{M}) \bar{\mathbf{q}} = \mathbf{0} \quad (23)$$

$$\mathbf{K} = \int_{\Omega} \begin{bmatrix} \mathbf{B}_{1l} \\ \mathbf{B}_{2l} \\ \mathbf{B}_{3l} \end{bmatrix}^T \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{G} \\ \mathbf{B} & \mathbf{D} & \mathbf{H} \\ \mathbf{G} & \mathbf{H} & \mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{B}_{1l} \\ \mathbf{B}_{2l} \\ \mathbf{B}_{3l} \end{bmatrix} d\Omega + \int_{\Omega} (\mathbf{B}_{sl})^T \mathbf{D}^s \mathbf{B}_{sl} d\Omega, \quad (24)$$

$$\mathbf{M} = \int_{\Omega} \begin{bmatrix} \mathbf{N}_{1l} \\ \mathbf{N}_{2l} \\ \mathbf{N}_{3l} \end{bmatrix}^T \begin{bmatrix} l_1 & l_2 & l_4 \\ l_2 & l_3 & l_5 \\ l_4 & l_5 & l_6 \end{bmatrix} \begin{bmatrix} \mathbf{N}_{1l} \\ \mathbf{N}_{2l} \\ \mathbf{N}_{3l} \end{bmatrix} d\Omega \quad (25)$$

$$\tilde{\mathbf{K}} = \mathbf{K} + \mathbf{p} (\mathbf{K}_A^p)^T \mathbf{K}_A^p \quad (26)$$

$$\left(\sum_{l=1}^{m \times n} \mathbf{R}_l(\xi, \eta) + \alpha \sum_{l=1}^{m \times n} \nabla^2 \mathbf{R}_l(\xi, \eta) \right) \varpi_{OA}^l = \mathbf{K}_A^p \mathbf{q}_A \quad (27)$$

3. Numerical results

3.1. Convergence and verification investigations

In this section, the proposed numerical model is evaluated through convergence and verification

studies. For all computational examples in this subsection, square plates constructed from three TPMS architectures (Primitive, Gyroid, and IWP) are considered. The structures are assumed to be made of SUS304 steel with $E_m = 200$ GPa,

$\nu_m = 0.3$, $\rho_m = 8000 \text{ kg/m}^3$. Furthermore, to facilitate comparison, the computed natural frequencies are expressed in a nondimensional form according to the formulation given in Eq. (28)

$$\omega = \sqrt[4]{\frac{12\omega^2 \rho_m a^4 (1 - \nu^2)}{h^2 E_m}}. \quad (28)$$

Before conducting comprehensive verification, a convergence study of the penalty parameter p is systematically carried out to ensure numerical stability and optimal accuracy. The convergence behavior is examined for thick plates with a length-to-thickness ratio of $a/h = 10$ under fully simply supported (SSSS) boundary

conditions, considering two porosity distributions (Sym. and Asym.).

Table 3 presents the variation of the first normalized natural frequency against penalty parameter values ranging from $p = 10^6$ to $p = 10^{15}$. It is clearly observed that the frequency values stabilize and converge rapidly as the penalty parameter increases, reaching a steady plateau at $p = 10^{13}$. At this optimal value, the computed frequencies also show excellent agreement with the reference solutions provided by Nguyen-Xuan [14]. Consequently, $p = 10^{13}$ is selected for all subsequent numerical computations in this study.

Table 3. Convergence of the first normalized natural frequencies with respect to the penalty parameter p (SSSS, $a/h = 10$).

Penalty parameter	Sym. porosity			Asym. porosity		
	P type	G type	IWP type	P type	G type	IWP type
$p = 10^6$	1.7216	1.7203	1.7235	1.7117	1.7127	1.7173
$p = 10^7$	2.7578	2.7477	2.7785	2.6579	2.6694	2.7112
$p = 10^8$	3.6699	3.632	3.6998	3.3656	3.3672	3.4570
$p = 10^9$	4.0927	4.0032	4.0803	3.6418	3.6069	3.7110
$p = 10^{10}$	4.1588	4.0583	4.1364	3.6817	3.6397	3.7456
$p = 10^{11}$	4.1682	4.0673	4.1455	3.6878	3.6448	3.7509
$p = 10^{12}$	4.1781	4.0771	4.1541	3.6972	3.6524	3.7570
$p = 10^{13}$	4.1990	4.0987	4.1732	3.7056	3.6626	3.7678
$p = 10^{14}$	4.2073	4.1088	4.1843	3.7072	3.6652	3.7713
$p = 10^{15}$	4.2084	4.1102	4.1860	3.7074	3.6655	3.7718
Nguyen-Xuan [14]	4.1928	4.0891	4.1490	3.7065	3.6638	3.7639

Based on the established optimal penalty parameter, we subsequently conduct a broader comparative study to verify the capability of the proposed model in capturing the free vibration response of TPMS plates. In this investigation, both fully simply supported (SSSS) and fully clamped (CCCC) boundary conditions are employed, and the length-to-thickness ratio is set to $a/h = 10$ for thick plates and $a/h = 200$ for thin plates.

Table 4 presents a comparison of the first normalized natural frequencies obtained from the present model against the four-variable IGA method reported in [14]. The comparison reveals highly consistent results, verifying the accuracy and reliability of the current formulation. Slight deviations observed in certain cases can be attributed to the inherent mathematical differences between the three-variable and four-variable plate theories, as well as the incorporation of the penalty

function.

3.2. Parametric investigation

This subsection examines the influence of various distribution functions on the free vibration characteristics of TPMS plates. Consequently, we analyze TPMS plates including three geometric configurations: Gyroid, Primitive, and IWP at varying length-to-thickness ratios. In addition, these structures are subjected to an SSSS boundary condition and a Uni. distribution along the thickness direction.

Table 5 presents the normalized first natural frequencies concerning various distribution

functions $f(z)$, TPMS types, and length-to-thickness ratios (a/h). As observed, when the a/h ratio reaches 100, the normalized natural frequencies show a minimal change, clustering around 3.6 for Primitive and IWP structures, and slightly lower at approximately 3.53 for Gyroid, suggesting a negligible influence of the distribution function for thin plates. In contrast, for thicker plates with an aspect ratio of 10, the natural frequency values exhibit a small variation across the distribution functions, with cases 1, 2, and 3 showing frequencies that are less than 4% lower than those of cases 4, 5, and 6.

Table 4. The first normalized natural frequencies of TPMS plates

Density Distribution	a/h	TPMS Type	SSSS		CCCC	
			Present	Nguyen-Xuan [14]	Present	Nguyen-Xuan [14]
Sym. porosity	200	Primitive	4.3172	4.3148	5.7535	5.7507
		Gyroid	4.2408	4.2384	5.6955	5.6928
		IWP	4.3184	4.3161	5.8659	5.8634
	10	Primitive	4.1990	4.1928	5.2585	5.3386
		Gyroid	4.0987	4.0891	5.1084	5.1814
		IWP	4.1732	4.1490	5.2056	5.2746
Asym. porosity	200	Primitive	3.7735	3.7735	4.9061	4.9064
		Gyroid	3.7455	3.7455	5.0208	5.0212
		IWP	3.8602	3.8603	5.2547	5.2551
	10	Primitive	3.7056	3.7065	4.6535	4.6924
		Gyroid	3.6626	3.6638	4.6935	4.7343
		IWP	3.7678	3.7639	4.8616	4.8999

Table 5. The effect of varying distribution functions and the a/h ratio on the vibration frequencies across different TPMS geometries.

a/h	TPMS type	The distribution function of $f(z)$					
		case 1	case 2	case 3	case 4	case 5	case 6
10	P	3.5369	3.5513	3.5528	3.5947	3.5835	3.5948
	G	3.4544	3.4744	3.4775	3.5502	3.5328	3.5503
	IWP	3.5107	3.5379	3.5435	3.6558	3.6301	3.6558
100	P	3.6004	3.6005	3.6005	3.6012	3.6011	3.6012
	G	3.5294	3.5295	3.5296	3.5306	3.5304	3.5306
	IWP	3.5996	3.5998	3.5998	3.6013	3.6010	3.6013

In the following numerical example, we examine how variations in the porosity coefficient and mass distribution patterns affect the natural frequencies of three TPMS-based plate structures.

Additionally, the impact of several boundary conditions, namely SSSS and CCCC, is also examined, with the length-to-thickness ratio established at $a/h = 10$. Fig. 2 shows a

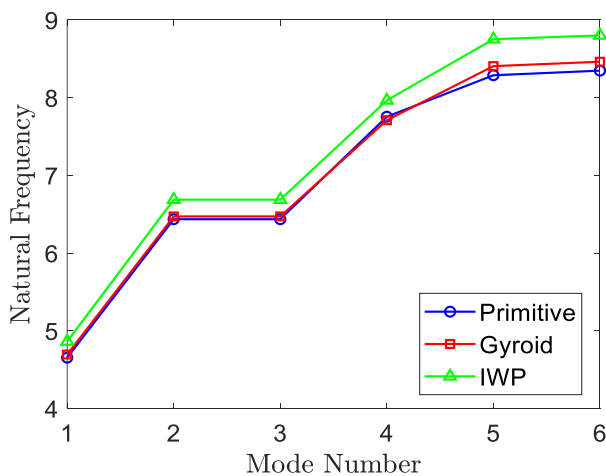
comparison of the first six natural frequencies for the three cases studied. The results reveal that among the TPMS architectures, plates constructed with the IWP structure exhibit the highest fundamental frequencies, whereas P-based plates show lower values, and G-based configurations yield the lowest frequencies.

Generally, the results presented in Table 6 demonstrate a strong dependency of the natural frequencies on the porosity distribution patterns. It is important to note that the mean relative mass ($M_R = 0.35$) and the maximum relative density ($R_{max} = 1$, as defined in Eq. (3)) of the plate are kept constant across all these investigations. Therefore, the variation in natural frequencies is governed exclusively by changes in the structural flexural stiffness. For the Sym. distribution, as the porosity coefficient increases, solid material is distributed towards the top and bottom outer surfaces, while the porosities are concentrated around the mid-plane. Because the outer layers contribute most significantly to the cross-sectional moment of inertia, this material distribution effectively enhances the overall bending stiffness without adding extra mass, thereby increasing the natural frequencies. Conversely, the Asym. distribution concentrates porosity at one of the outer surfaces. This specific material allocation severely degrades the flexural stiffness of the plate. Consequently, this stiffness reduction, coupled with

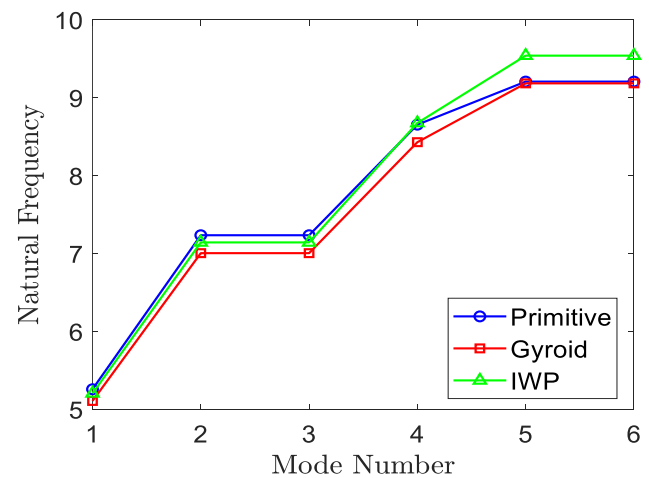
the constant mass, leads to a monotonic decrease in the natural frequencies as the porosity coefficient increases.

In addition, the CCCC boundary condition always results in higher natural frequencies compared to other boundary conditions for all porosity coefficients and distribution types. Physically, the fully clamped (CCCC) edges strongly restrict the plate from moving and rotating. These strong boundary constraints greatly increase the overall bending stiffness of the plate. Since the total mass remains unchanged, this higher stiffness directly leads to the highest natural frequencies.

Besides, the specific TPMS architecture (Primitive, Gyroid, IWP) significantly affects the frequency response. Among the three structures, the IWP type demonstrates the highest frequency values for both porosity levels and across all distribution types under the same boundary condition. This result comes from the internal geometric shapes of the TPMS architectures. At the same relative density, the IWP structure creates a stronger and better-connected network than the Primitive and Gyroid structures. Therefore, the IWP structure provides higher effective elastic and shear moduli. This better material stiffness makes the whole plate stiffer, which produces higher natural vibration frequencies.

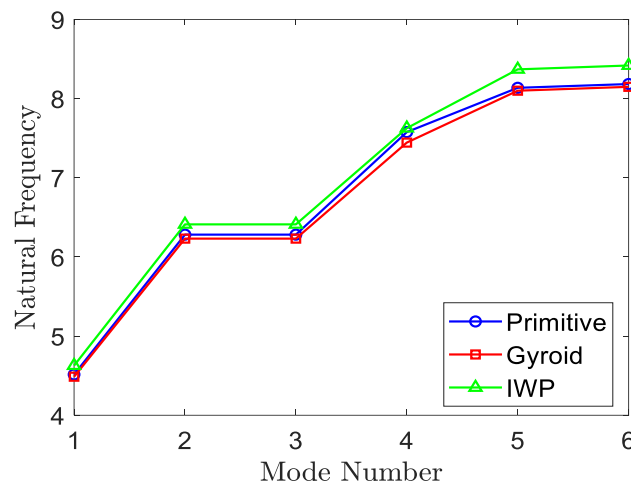


(a) Asym. distribution



(b) Sym. distribution

Fig. 2. The first six natural frequencies for three porosity distributions



(c) Uni. distribution

Fig. 2. (continued)**Table 6.** Influence of porosity coefficient and distribution on fundamental frequencies of TPMS plates

BC Type	TPMS Type	$R_0 = 0.75$			$R_0 = 0.8$		
		Asym.	Sym.	Uni.	Asym.	Sym.	Uni.
SSSS	P	3.7056	4.1990	3.5528	3.5935	4.3140	3.5528
	G	3.6626	4.0987	3.4775	3.5257	4.1894	3.4775
	IWP	3.7678	4.1732	3.5435	3.6560	4.2651	3.5435
CCCC	P	4.6535	5.2585	4.5146	4.5267	5.3928	4.5146
	G	4.6935	5.1084	4.4868	4.5545	5.2175	4.4868
	IWP	4.8616	5.2056	4.6272	4.7568	5.3128	4.6272
SFSF	P	3.4550	3.8562	3.3159	3.3576	3.9568	3.3159
	G	3.3310	3.7363	3.1724	3.2028	3.8258	3.1724
	IWP	3.3433	3.7591	3.1789	3.2804	3.8514	3.1789
CFCF	P	3.6067	4.1963	3.5056	3.4929	4.3284	3.5056
	G	3.7399	4.1061	3.5514	3.6198	4.2111	3.5514
	IWP	3.9140	4.2057	3.7157	3.8238	4.3006	3.7157
BC Type	TPMS Type	$R_0 = 0.85$			$R_0 = 0.9$		
		Asym.	Sym.	Uni.	Asym.	Sym.	Uni.
SSSS	P	3.4552	4.3694	3.5528	3.3069	4.3849	3.5528
	G	3.3619	4.2163	3.4775	3.1942	4.2054	3.4775
	IWP	3.5160	4.2945	3.5435	3.3637	4.2852	3.5435
CCCC	P	4.3724	5.4739	4.5146	4.2076	5.5142	4.5146
	G	4.3776	5.2859	4.4868	4.1874	5.3147	4.4868
	IWP	4.6118	5.3869	4.6272	4.4433	5.4349	4.6272
SFSF	P	3.2234	4.0095	3.3159	3.0870	4.0299	3.3159
	G	3.0156	3.8605	3.1724	2.9490	3.8622	3.1724
	IWP	3.1489	3.8897	3.1789	3.0080	3.8941	3.1789
CFCF	P	3.3618	4.4103	3.5056	3.2287	4.4499	3.5056
	G	3.4674	4.2687	3.5514	3.3064	4.2748	3.5514
	IWP	3.6982	4.3561	3.7157	3.5532	4.3718	3.7157

4. Conclusion

This paper investigated the free vibration responses of TPMS plates with different geometric configurations using a three-variable HSDT combined with the IGA. Furthermore, we explored the influence of input parameters such as porosity coefficient, length-to-thickness ratio, and different boundary conditions, with a particular focus on the effect of different distribution functions on the natural frequency of TPMS plates. Based on the obtained results, the following key conclusions can be drawn:

The proposed three-variable IGA model shows good agreement with existing methods for predicting the free vibration of TPMS plates. Theoretically, this approach reduces computational costs by lowering the degrees of freedom. However, future experimental validations and quantitative CPU time comparisons are necessary to confirm these efficiency gains.

The results indicate a significant dependency of vibration frequency on porosity distribution. While increasing porosity reduces the frequency of Asym. and Uni. types, it leads to an opposite trend for the Sym. configuration.

Finally, considering all the investigation results, the IWP-based TPMS plate demonstrates a higher average vibration frequency compared to the other two geometric configurations. This observation plays a significant role in choosing the most effective TPMS type for subsequent investigations into vibration frequency analysis of plates.

Based on the analysis results in this study, we recognize the need for further research into utilizing this numerical method for TPMS materials reinforced with high-stiffness and high-strength materials such as graphene platelets or carbon nanotubes. It is recommended that future studies extend beyond free vibration analysis to encompass phenomena related to structural instability and the energy dissipation characteristics of TPMS plates.

References

- [1] D.W. Abueidda, R.K.A. Al-Rub, A.S. Dalaq, D.-W. Lee, K.A. Khan, I. Jasiuk. (2016). Effective conductivities and elastic moduli of novel foams with triply periodic minimal surfaces. *Mechanics of Materials*, 95, 102-115. <https://doi.org/10.1016/j.mechmat.2016.01.004>
- [2] P. Tran, C. Peng. (2021). Triply periodic minimal surfaces sandwich structures subjected to shock impact. *Journal of Sandwich Structures & Materials*, 23(6), 2146-2175. <https://doi.org/10.1177/1099636220905551>
- [3] T.D. Ngo, A. Kashani, G. Imbalzano, K.T.Q. Nguyen, D. Hui. (2018). Additive manufacturing (3D printing): A review of materials, methods, applications and challenges. *Composites Part B: Engineering*, 143, 172-196. <https://doi.org/10.1016/j.compositesb.2018.02.012>
- [4] I. Maskery, L. Sturm, A.O. Aremu, A. Panesar, C.B. Williams, C.J. Tuck, R.D. Wildman, I.A. Ashcroft, R.J.M. Hague. (2018). Insights into the mechanical properties of several triply periodic minimal surface lattice structures made by polymer additive manufacturing. *Polymer*, 152, 62-71. <https://doi.org/10.1016/j.polymer.2017.11.049>
- [5] O. Al-Ketan, R. Rowshan, R.K.A. Al-Rub. (2018). Topology-mechanical property relationship of 3D printed strut, skeletal, and sheet based periodic metallic cellular materials. *Additive Manufacturing*, 19, 167-183. <https://doi.org/10.1016/j.addma.2017.12.006>
- [6] N.V. Nguyen, K.Q. Tran, P. Phung-Van, J. Lee, H. Nguyen-Xuan. (2023). An isogeometric analysis of functionally graded triply periodic minimal surface microplates. *Aerospace Science and Technology*, 137, 108270. <https://doi.org/10.1016/j.ast.2023.108270>
- [7] T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs. (2005). Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement. *Computer Methods in Applied Mechanics and*

- Engineering*, 194(39-41), 4135-4195. <https://doi.org/10.1016/j.cma.2004.10.008>
- [8] H.-Q. Tran, V.-T. Vu. (2026). Free vibration and buckling analysis of bidirectional functionally graded plates with integrated piezoelectric layers. *Journal of Science and Transport Technology*, 6(1), 48-66. <https://doi.org/10.58845/jstt.utt.2026.en.6.1.48-66>
- [9] N.T. Phuong, V.M. Duc, L.N. Quang. (2024). Nonlinear dynamic buckling analysis of the FG-GPLRC cylindrical and sinusoid panels with porous core under time-dependent axial compression. *Journal of Science and Transport Technology*, 4(1), 13-22. <https://doi.org/10.58845/jstt.utt.2024.en.4.1.13-22>
- [10] J.A. Cottrell, A. Reali, Y. Bazilevs, T.J.R. Hughes. (2006). Isogeometric analysis of structural vibrations. *Computer Methods in Applied Mechanics and Engineering*, 195(41-43), 5257-5296. <https://doi.org/10.1016/j.cma.2005.09.027>
- [11] S. Shojaee, N. Valizadeh, E. Izadpanah, T. Bui, T.-V. Vu. (2012). Free vibration and buckling analysis of laminated composite plates using the NURBS-based isogeometric finite element method. *Composite Structures*, 94(5), 1677-1693. <https://doi.org/10.1016/j.compstruct.2012.01.012>
- [12] H.-T. Thai, S.-E. Kim. (2015). A review of theories for the modeling and analysis of functionally graded plates and shells. *Composite Structures*, 128, 70-86. <https://doi.org/10.1016/j.compstruct.2015.03.010>
- [13] K.M. Liew, X. Zhao, A.J.M. Ferreira. (2011). A review of meshless methods for laminated and functionally graded plates and shells. *Composite Structures*, 93(8), 2031-2041. <https://doi.org/10.1016/j.compstruct.2011.02.018>
- [14] H. Nguyen-Xuan, K.Q. Tran, C.H. Thai, J. Lee. (2023). Modelling of functionally graded triply periodic minimal surface (FG-TPMS) plates. *Composite Structures*, 315, 116981. <https://doi.org/10.1016/j.compstruct.2023.116981>
- [15] V. Khalafi, J. Fazilati. (2018). Supersonic panel flutter of variable stiffness composite laminated skew panels subjected to yawed flow by using NURBS-based isogeometric approach. *Journal of Fluids and Structures*, 82, 198-214. <https://doi.org/10.1016/j.jfluidstructs.2018.07.002>
- [16] I. Mechab, B. Mechab, S. Benaissa. (2013). Static and dynamic analysis of functionally graded plates using four-variable refined plate theory by the new function. *Composites Part B: Engineering*, 45(1), 748-757. <https://doi.org/10.1016/j.compositesb.2012.07.015>
- [17] L.V. Tran, C.H. Thai, H.T. Le, B.S. Gan, J. Lee, H. Nguyen-Xuan. (2014). Isogeometric analysis of laminated composite plates based on a four-variable refined plate theory. *Engineering Analysis with Boundary Elements*, 47, 68-81. <https://doi.org/10.1016/j.enganabound.2014.05.013>
- [18] Q.H. Nguyen, L.B. Nguyen, H.B. Nguyen, H. Nguyen-Xuan. (2020). A three-variable high order shear deformation theory for isogeometric free vibration, buckling and instability analysis of FG porous plates reinforced by graphene platelets. *Composite Structures*, 245, 112321. <https://doi.org/10.1016/j.compstruct.2020.112321>
- [19] T.N. Nguyen, T.D. Ngo, H. Nguyen-Xuan. (2017). A novel three-variable shear deformation plate formulation: theory and Isogeometric implementation. *Computer Methods in Applied Mechanics and Engineering*, 326, 376-401. <https://doi.org/10.1016/j.cma.2017.07.024>

- [20] T. Horger, A. Reali, B. Wohlmuth, L. Wunderlich. (2019). A hybrid isogeometric approach on multi-patches with applications to Kirchhoff plates and eigenvalue problems. *Computer Methods in Applied Mechanics and Engineering*, 348, 396-408. <https://doi.org/10.1016/j.cma.2018.12.038>
- [21] G. Guarino, P. Antolin, A. Milazzo, A. Buffa. (2024). An interior penalty coupling strategy for isogeometric non-conformal Kirchhoff–Love shell patches. *Engineering with Computers*, 40, 3031-3057. <https://doi.org/10.1007/s00366-024-01965-5>
- [22] N.T. Phuong, V.H. Nam, B.T. Tu. (2025). An analytical approach for nonlinear thermo-mechanical buckling behavior of Porous FG-GPLRC circular plates and spherical caps. *Journal of Science and Transport Technology*, 5(1), 40-60. <https://doi.org/10.58845/jstt.utt.2025.en.5.1.40-60>
- [23] L.N. Quang, V.M. Duc, T.Q. Minh. (2024). Nonlinear buckling and postbuckling behaviors of porous FG-GPLRC cylindrical shells with stiffeners subjected to external pressure. *Journal of Science and Transport Technology*, 4(2), 24-36. <https://doi.org/10.58845/jstt.utt.2024.en.4.2.24-36>
- [24] I. Maskery, A.O. Aremu, L. Parry, R.D. Wildman, C.J. Tuck, I.A. Ashcroft. (2018). Effective design and simulation of surface-based lattice structures featuring volume fraction and cell type grading. *Materials & Design*, 155, 220-232. <https://doi.org/10.1016/j.matdes.2018.05.058>
- [25] D. Li, W. Liao, N. Dai, G. Dong, Y. Tang, Y.M. Xie. (2018). Optimal design and modeling of gyroid-based functionally graded cellular structures for additive manufacturing. *Computer-Aided Design*, 104, 87-99. <https://doi.org/10.1016/j.cad.2018.06.003>
- [26] N.V. Nguyen, K.Q. Tran, D.T.T. Do, C.H. Thai, K.K. Žur, H. Nguyen-Xuan. (2024). An isogeometric analysis of solar panels with a bio-inspired substrate. *Engineering Analysis with Boundary Elements*, 166, 105854. <https://doi.org/10.1016/j.enganabound.2024.105854>
- [27] K.Q. Tran, T.-D. Hoang, J. Lee, H. Nguyen-Xuan. (2024). Three novel computational modeling frameworks of 3D-printed graphene platelets reinforced functionally graded triply periodic minimal surface (GPLR-FG-TPMS) plates. *Applied Mathematical Modelling*, 126, 667-697. <https://doi.org/10.1016/j.apm.2023.10.043>
- [28] C.H. Thai, P.T. Hung, H. Nguyen-Xuan, P. Phung-Van. (2024). A meshfree method for functionally graded triply periodic minimal surface plates. *Composite Structures*, 332, 117913. <https://doi.org/10.1016/j.compstruct.2024.117913>
- [29] H. Tang, N.V. Nguyen, H. Nguyen-Xuan, J. Lee. (2024). Isogeometric 3D optimal designs of functionally graded triply periodic minimal surface plates. *International Journal of Mechanical Sciences*, 277, 109406. <https://doi.org/10.1016/j.ijmecsci.2024.109406>
- [30] T.N. Nguyen, N. Wattanasakulpong, N.P. Nguyen, P. Fakharian, S. Eiadtrong. (2025). Isogeometric analysis of functionally graded triply periodic minimal surface shells. *Mechanics of Advanced Materials and Structures*, 32(21), 5447-5460. <https://doi.org/10.1080/15376494.2024.2423278>
- [31] H.X. Nguyen, T.N. Nguyen, M. Abdel-Wahab, S.P.A. Bordas, H. Nguyen-Xuan, T.P. Vo. (2017). A refined quasi-3D isogeometric analysis for functionally graded microplates based on the modified couple stress theory. *Computer Methods in Applied Mechanics and Engineering*, 313, 904-940. <https://doi.org/10.1016/j.cma.2016.10.002>

- [32] J.N. Reddy. (1984). A simple higher-order theory for laminated composite plates. *Journal of Applied Mechanics*, 51(4), 745-752. <https://doi.org/10.1115/1.3167719>
- [33] H. Arya, R.P. Shimpi, N.K. Naik. (2002). A zigzag model for laminated composite beams. *Composite Structures*, 56(1), 21-24. [https://doi.org/10.1016/S0263-8223\(01\)00178-7](https://doi.org/10.1016/S0263-8223(01)00178-7)
- [34] H. Nguyen-Xuan, L.V. Tran, C.H. Thai, S. Kulasegaram, S.P.A. Bordas. (2014). Isogeometric analysis of functionally graded plates using a refined plate theory. *Composites Part B: Engineering*, 64, 222-234. <https://doi.org/10.1016/j.compositesb.2014.04.001>
- [35] R.P. Shimpi, & H.G. Patel. (2006). A two-variable refined plate theory for orthotropic plate analysis. *International Journal of Solids and Structures*, 43(22-23), 6783-6799. <https://doi.org/10.1016/j.ijsolstr.2006.02.007>
- [36] A.M. Zenkour. (2013). A simple four-unknown refined theory for bending analysis of functionally graded plates. *Applied Mathematical Modelling*, 37(20-21), 9041-9051. <https://doi.org/10.1016/j.apm.2013.04.022>
- [37] D.W. Pepper, J.C. Heinrich. (2005). The Finite Element Method: Basic Concepts and Applications. 2nd Edition. *CRC Press*. <https://doi.org/10.1201/9780203942352>
- [38] J.N. Reddy. (2005). An introduction to the finite element method (3rd Edition). *McGraw-Hill*.