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Numerical study of punching shear behavior of flat slab with openings

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Abstract: Punching shear is a critical failure mode in reinforced concrete flat slabs, particularly when openings are located near slab–column connections, where they disrupt load transfer and intensify stress concentration. This study develops a validated nonlinear finite element model to investigate the punching shear behaviour of flat slabs with openings. The model shows good agreement with experimental results in terms of load–deflection response and failure mode, with discrepancies in ultimate load below 4%. A systematic parametric study is conducted to quantify the effects of opening location, size, and shape. The results show that openings adjacent to the column reduce the ultimate load to 166.7 kN, while increasing the distance to 50 mm and 100 mm enhances capacity by 23.86% and 24.93%, respectively. Increasing opening size leads to strength reductions of up to 12.4%, whereas circular openings exhibit higher capacity than square ones due to reduced stress concentration. The study provides a validated numerical framework and a quantitative assessment of the combined influence of opening geometry and position on punching shear behaviour, offering new insight into the governing failure mechanisms.

Keywords: Flat slabs; opening; punching shear; numerical study.

1. Introduction

Nowadays, Reinforced Concrete (RC) flat slabs are widely applied in modern construction thanks to their ability to create flexible open spaces, reduce the overall building height, and increase the clear height of floors. In addition, due to the simple formwork and reinforcement installation process, the construction time is shortened, thereby reducing the overall construction cost.

The absence of beams allows greater freedom in architectural layout and facilitates the installation of mechanical, electrical, and plumbing (MEP) systems. However, the slab–column connection in flat slab systems is susceptible to punching shear failure. Punching failure is typically brittle and may occur suddenly without significant warning, which makes the accurate prediction of punching shear capacity a critical issue in structural design.

During construction and operation, creating openings in slabs is often unavoidable to serve various purposes. Such openings are typically required for fire protection piping systems, water supply and drainage systems, heating ducts, ventilation, and air-conditioning. These openings are typically located next to the columns (Fig. 1). In some cases, large openings may be needed to connect to public utilities, such as exhaust pipes, gas ducts, or to serve other architectural purposes, such as admitting natural light, creating vertical voids, or accommodating emergency staircases (Fig. 2). Openings may even be created after the structure is completed to accommodate the installation of various systems in the existing structure.

Fig. 1. Circular openings adjacent to the column

Fig. 2. Large opening in flat slab [1]

Although these openings offer practical benefits, their presence significantly affects the structural performance of the slab, especially its punching shear strength at slab - column connections. Openings cause discontinuities in both concrete and reinforcement, leading to a considerable decrease in punching shear strength and making the slab more susceptible to punching shear failure. Punching failure often occurs in a brittle manner, with minimal deflection followed by a near - total loss of load-bearing capacity. It typically manifests as a conical or pyramid-shaped crack around the column head, which may lead to the collapse of the entire structure, as shown in Fig. 3.

The punching shear behavior of reinforced concrete (RC) flat slabs with openings has been the subject of numerous investigations, focusing on parameters namely the location, number, size and shape of openings [2-12]. Specifically, Ö. Mercimek et al. [2] reported that openings next to the column reduced punching shear strength by up to 68%, while increasing the number of openings from one to two reduced strength by about 11%, and enlarging openings from 300 × 300 mm to 500 × 500 mm caused a 22% reduction. Similarly, Tomas Augustin et al. [3] further showed that openings near columns decreased the critical shear perimeter and slab stiffness by up to 64%. E. Yooprasertchai et al. [4] concluded that circular openings were least detrimental, while rectangular openings produced the greatest reduction due to stress concentration at corners. El-Shafiey et al. [5, 6] investigated reinforced concrete flat slabs with various opening configurations located near the column region. Their results showed that openings positioned close to the column significantly reduce punching shear capacity due to the reduction of the effective shear transfer zone around the column (up to 51%). Similar observations were reported in the experimental work of Liberati et al. [7], where the presence of openings altered both the crack propagation pattern and the ultimate punching capacity of slab specimens. In addition, Anil et al. [8] conducted an experimental investigation on reinforced concrete flat slabs containing different opening configurations and reported that the reduction in punching strength strongly depends on the distance between the opening and the column as well as the size of the opening. These experimental results demonstrated that openings located within the critical punching region may significantly modify the stress distribution and failure mechanism of slab–column connections.

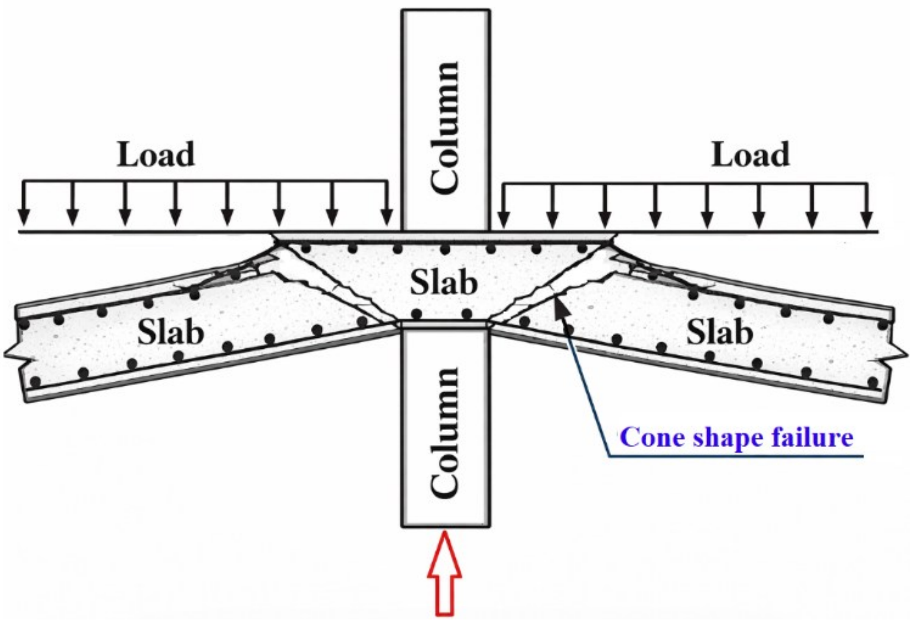


Fig. 3. Punching failures in flat slab.

In addition to experimental investigations, several studies have utilized finite element method (FEM) simulations to analyze RC flat slabs with openings to understand the structural behaviour of this structure [13-16]. With the development of computational mechanics, nonlinear finite element analysis has become an effective tool for simulating the complex behaviour of reinforced concrete structures, including cracking, stiffness degradation, and failure mechanisms. Numerical modelling enables researchers to investigate the influence of various parameters that may be difficult to examine experimentally. Several researchers have employed commercial FE software packages such as ABAQUS and ANSYS to model reinforced concrete slabs with openings and to evaluate their punching resistance. For example, E. Ö. Köroğlu et al. [13] used ANSYS to predict the ultimate punching shear stress, while Balomenos et al. [14] employed ABAQUS for similar simulations. Both studies reported close agreement between simulated and experimental load–displacement curves and failure modes, indicating that numerical simulations can be effectively applied to study the behavior of reinforced concrete flat slabs with openings. Such simulations save considerable time and cost by eliminating the need for specimen fabrication; adjusting model parameters allows the prediction of structural response under various conditions.

Despite the significant progress achieved in both experimental and numerical studies, several aspects of punching shear failure in slabs with openings remain insufficiently understood. Experimental investigations often involve a limited number of specimens due to cost and practical constraints, which restricts the ability to systematically investigate the influence of different parameters. Analytical models provide simplified predictions but may not fully capture the complex stress redistribution and crack development occurring around openings. Therefore, validated nonlinear finite element modelling can provide an effective approach for further investigating the punching behaviour of flat slabs with openings.

In particular, the mechanical influence of opening size and opening location on the stress distribution, crack propagation, and ultimate punching capacity of slab–column connections require further clarification through numerical simulations. Reliable numerical models must first be validated against experimental results before they can be used for parametric investigations aimed at improving the understanding of punching failure mechanisms.

Therefore, the objective of this study is to develop a three-dimensional nonlinear finite element model (Cast3M software) to investigate the punching shear behaviour of reinforced concrete flat slabs with openings. The numerical model is implemented using the Cast3M finite element platform and incorporates a damage-based constitutive model for concrete. The model is first validated against experimental results reported by El-Shafiey et al. [5] to verify its ability to reproduce load–deflection response, crack patterns, and ultimate punching capacity. After validation, the numerical model is employed to conduct a parametric study investigating the influence of opening size and opening location on the punching shear behaviour of flat slabs. The results provide additional insight into the mechanical mechanisms governing punching failure in slab–column connections with openings.

2. Finite element modeling

2.1. Model development

To validate the FEM model, experimental results from El-Shafiey et al. [5] were employed. The tested specimen was a square slab measuring 1700 × 1700 mm with a thickness of 120 mm, supported on all four edges with simple line supports. A centrally located reinforced concrete column of 150 × 150 mm cross-section and 300 mm height was monolithically cast with the slab. Axial compressive loading was applied through the top surface of the column. Two cases were considered: a solid slab without openings (SC slab) and a slab with a 200 × 200 mm square opening adjacent to the column edge (S-SO-200-0), as illustrated in Fig. 4.

It should be noted that the validation of the numerical model is based on a limited number of experimental specimens. However, the selected cases are considered representative benchmark configurations, including a reference slab without openings and a slab with an opening located directly adjacent to the column, which corresponds to one of the most critical scenarios in terms of punching shear capacity reduction. These two cases allow the model to be evaluated under both undisturbed and severely disturbed stress conditions.

Therefore, although the number of validation samples is limited, they provide a reasonable and meaningful basis for assessing the capability of the numerical model to reproduce the global structural response and failure mechanism associated with punching shear in slabs with openings.

The reinforcement consisted of D12 bars at 100 mm spacing in both directions at the bottom (ρ = 1.13%) and D6 bars at 100 mm spacing at the top (ρ = 0.28%), while the column was reinforced with 4D12 longitudinal bars and D6 stirrups at 75 mm spacing. The concrete compressive strength at 28 days was 24 MPa [5].

In this study, the punching shear behavior of reinforced concrete (RC) flat slabs with openings was investigated using the finite element method (FEM). Numerical simulations were carried out with the software Cast3M [17], developed by the French Alternative Energies and Atomic Energy Commission (CEA). Eight-node hexahedral elements (CUB8) with three translational degrees of freedom per node in the Ux, Uy, and Uz directions were employed to model the concrete. The reinforcement was represented using two-node linear beam elements (SEG2), each node likewise possessing three translational degrees of freedom in the Ux, Uy, and Uz directions. The bond between the slab concrete and reinforcement was assumed to be perfectly bonded.

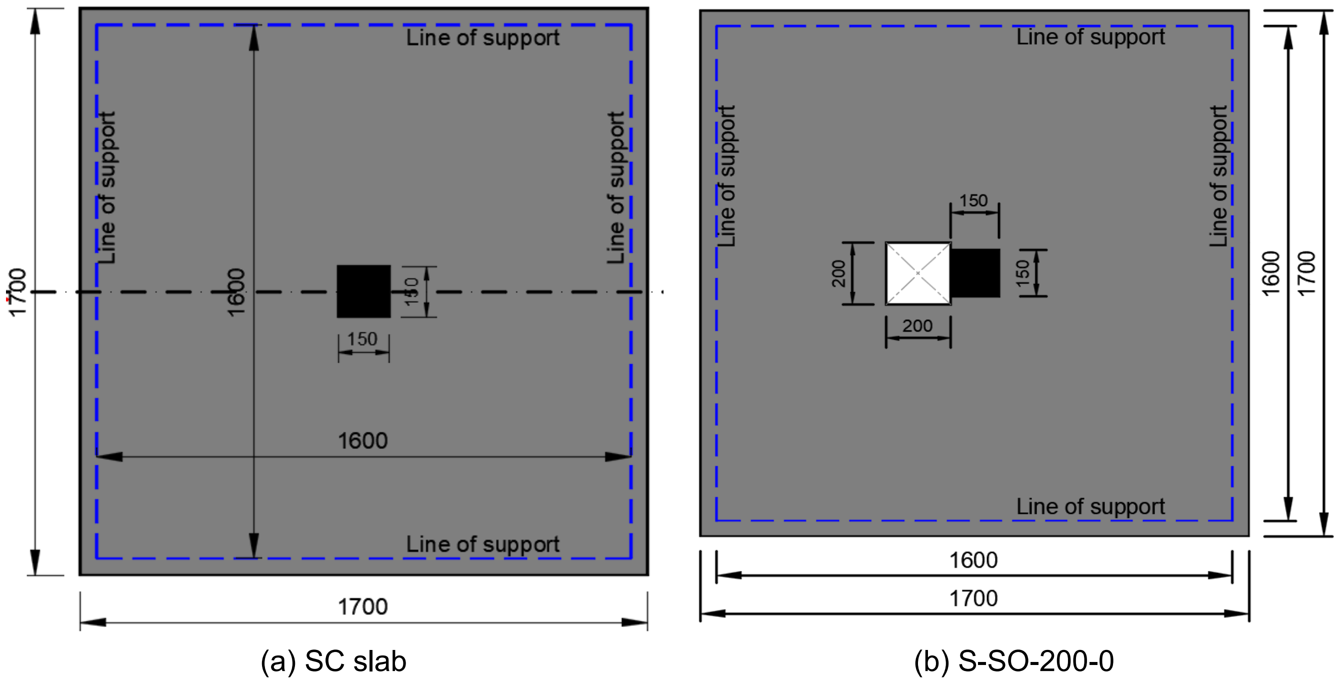


Fig. 4. Layout of the slab in this study. (a) SC slab, (b) S-SO-200-0 [5].

A mesh convergence study was conducted to determine an appropriate element size for the finite element simulations. In this study, the slab model was discretized using element sizes of 45 mm, 35 mm, 25 mm and 15 mm. The corresponding numerical analyses were performed and the resulting load–displacement curves are presented in Fig. 5.

All FE simulations were performed on a Dell Precision workstation equipped with an Intel Core i7-10850H CPU (6 cores, 12 threads, 2.70–5.10 GHz), an NVIDIA Quadro T2000 GPU (4 GB GDDR6), and 32 GB DDR4 RAM. The computational time per simulation ranged from approximately 60 to 120 minutes, depending on the model complexity and mesh density.

The results show that the load–displacement response converges when the element size reaches 25 mm. Further refinement of the mesh leads to only negligible changes in the predicted response while significantly increasing the computational cost. Therefore, an element size of 25 mm was adopted for all numerical simulations in the present study, providing a good balance between computational efficiency and numerical accuracy.

The elements of the S-SO-200-0 specimen are illustrated in Fig. 6. The supports were modeled as hinged connections. At the left support, the Uy and Uz degrees of freedom were constrained, while at the right support only the Uz degree of freedom was fixed. Similarly, at the bottom support, both Uy and Uz degrees of freedom were restrained, while at the top support only the Uz degree of freedom was constrained.

The accurate representation of concrete behaviour is essential for the reliable prediction of punching shear response in reinforced concrete slabs. In this present study, the nonlinear behaviour of concrete was modelled using the Mazars damage model [18], available within the Cast3M material library. This model is based on continuum damage mechanics and introduces a scalar damage variable to represent the progressive degradation of material stiffness due to microcracking.

Compared with plasticity-based models, such as the Concrete Damaged Plasticity (CDP) model [19], the Mazars model provides a more physically consistent representation of quasi-brittle materials like concrete. While plasticity models assume irreversible plastic deformation governed by a yield surface, the Mazars model directly captures stiffness degradation caused by distributed microcracking, which is the dominant failure mechanism in concrete under tensile and shear-dominated loading conditions. Furthermore, plasticity-based models typically require a large number of material parameters and involve complex return-mapping algorithms, which may lead to numerical instability and convergence difficulties in nonlinear finite element analysis.

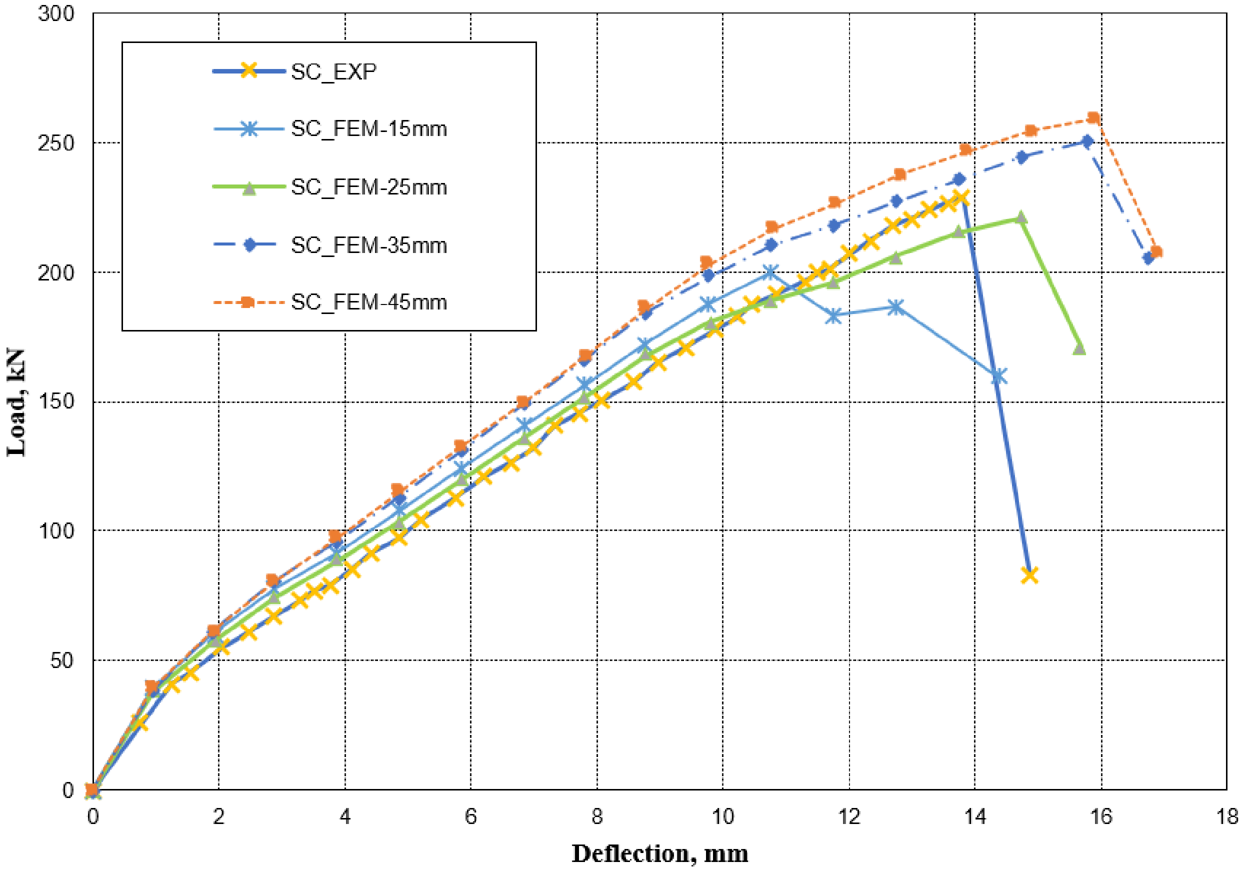


Fig. 5. Compare force-deflection with elements of different sizes.

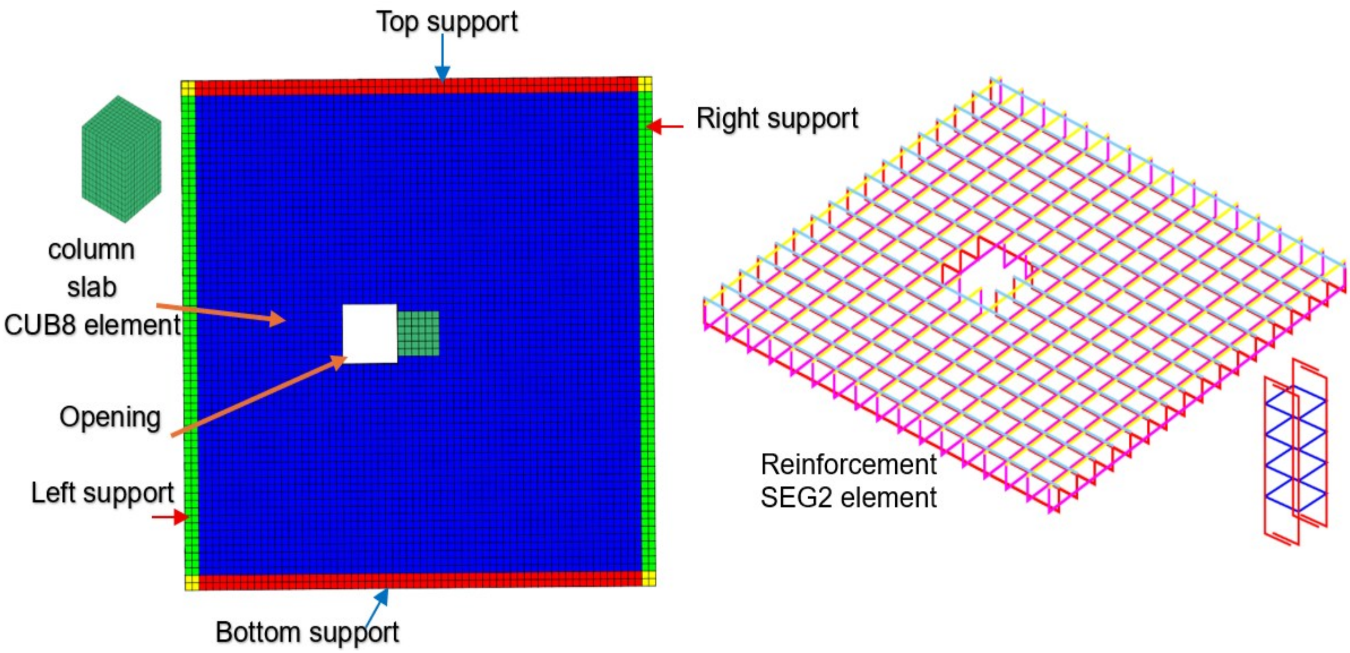


Fig. 6. Elements in finite model.

In contrast, the Mazars damage model employs a relatively simple formulation with a limited number of material parameters and does not require iterative plastic correction procedures. This results in improved numerical robustness and computational efficiency, which is particularly advantageous for large-scale parametric studies involving multiple finite element simulations.

Compared with crack-based approaches, such as smeared crack or discrete crack models, the Mazars model does not require explicit tracking of crack initiation and propagation paths. Instead, cracking is represented implicitly through the evolution of a scalar damage variable, allowing for a natural description of distributed cracking within the material. This feature is particularly beneficial in problems involving complex crack patterns, such as punching shear failure in flat slabs with openings.

In this model, the stress–strain relationship of concrete is expressed by incorporating a damage variable into the elastic constitutive law. As loading increases, microcracks develop within the material, leading to stiffness degradation that is represented by the evolution of the damage parameter d (d from 0 to 1), where d = 0 denotes the intact material state and d = 1 corresponds to a fully broken material. The parameter d accounts for the progressive deterioration of the material stiffness during damage evolution. The relationship between the damaged stiffness matrix $\underline{\underline{\Lambda}}_d$ and the undamaged stiffness matrix $\underline{\underline{\Lambda}}$ is expressed as:

$$\underline{\underline{\Lambda}}_d = \underline{\underline{\Lambda}}(1-d) \tag{1}$$

The Mazars parameters include $A_t, B_t, A_c, B_c, \varepsilon_{do}, k$, where ε_{do} is the tensile strain threshold for damage initiation, typically ranging from 0.5x10-4 to 1.5x10-4. Coefficients A and B define the post-peak softening rate, with higher values indicating steeper stiffness degradation. A_c, B_c describe compressive behavior, value of A_c ranging from 1 to 2 and B_c from 1000 to 2000. Parameters A_t and B_t describe tensile behavior, with A_t typically ranging from 0.7–1 and B_t from 9000–21000.

In Cast3M, the IDENTI command is used to define the required input properties: elastic modulus (E_c), compressive strength (f_c'), tensile strength (f_r), Poisson's ratio (υ), strain at peak compressive stress (ε_o) and residual tensile strength $f_{residual}$ (assuming 0.1 tensile strength of concrete). The IDENTI function is used to define material properties in the input file, and its syntax generally follows this structure: " ε_{do} A_t B_t A_c B_c= IDENTI Mazars $E_c \upsilon f_r$ $f_{residual}$ f_c' ε_o. The above values were determined based on the concrete compressive strength in accordance with the provisions of ACI 318-19 [20], as follows:

$$E_c=4700\sqrt{f_c'} \tag{2}$$

$$\varepsilon_0=\frac{2\sqrt{f_c'}}{E_c} \tag{3}$$

$$f_r=0.623\sqrt{f_c'} \tag{4}$$

Although the model uses a scalar damage variable, the formulation is capable of representing the nonlinear mechanical behaviour of concrete under complex stress states. So, it has been widely applied to simulate the nonlinear behaviour of reinforced concrete structures subjected to complex stress states. Previous studies [18], [20-23] have demonstrated that the Mazars damage model is capable of reproducing crack initiation, stiffness degradation, and failure mechanisms in concrete structures subjected to shear-dominated loading conditions.

Therefore, the Mazars damage model is considered suitable for the present study, where the punching shear behaviour of flat slabs is governed primarily by distributed cracking and stiffness degradation in the slab–column connection region.

In the present study, for the sake of simplicity, an elastic–perfectly plastic model is used for reinforcement. This simplified model, characterized by a bilinear relationship, is defined by two parameters: the elastic modulus and the yield tensile strength of the reinforcement and stirrup (Fig. 7).

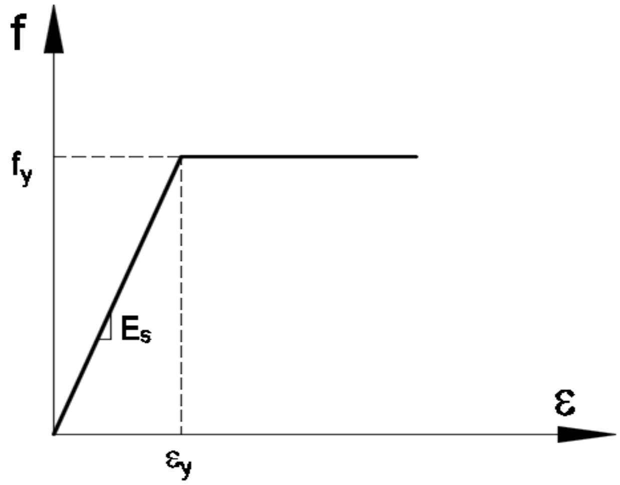


Fig. 7. Stress – strain of reinforcement and stirrup.

Table 1. Comparison of results of experimental and simulated slabs.

Specimens	Punching shear Pu (kN)		
	EXP	FEM	Error
SC slab	229.39	220.98	3.8 %
S-SO-200-0	172.49	166.73	3.5 %

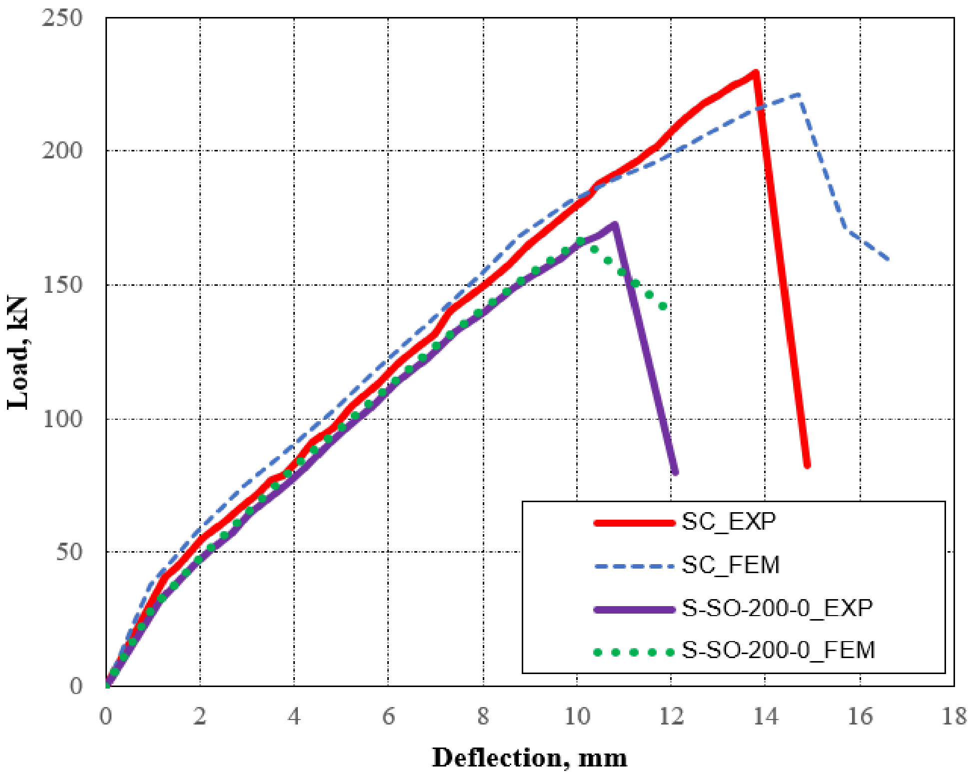
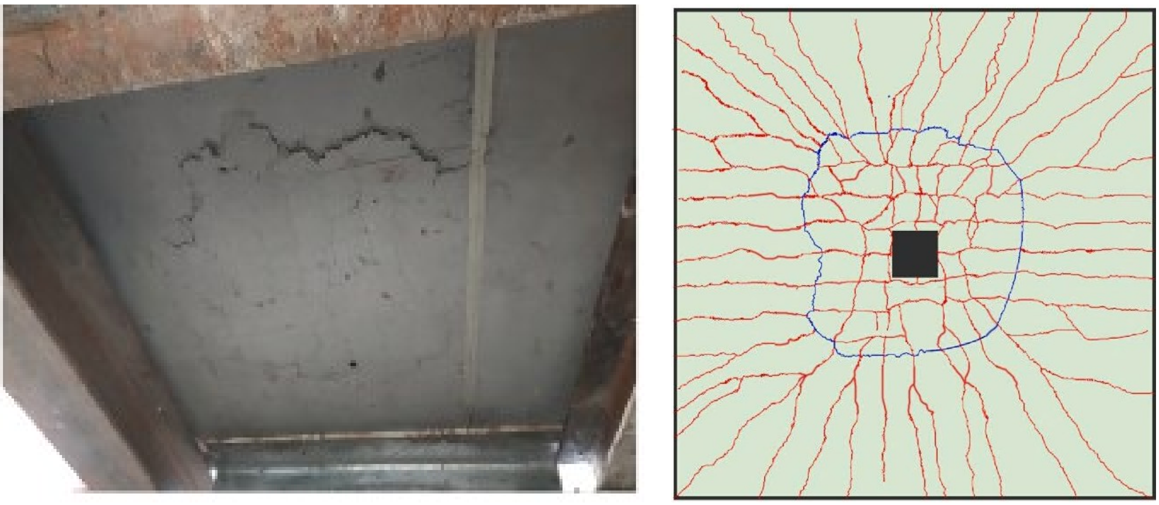


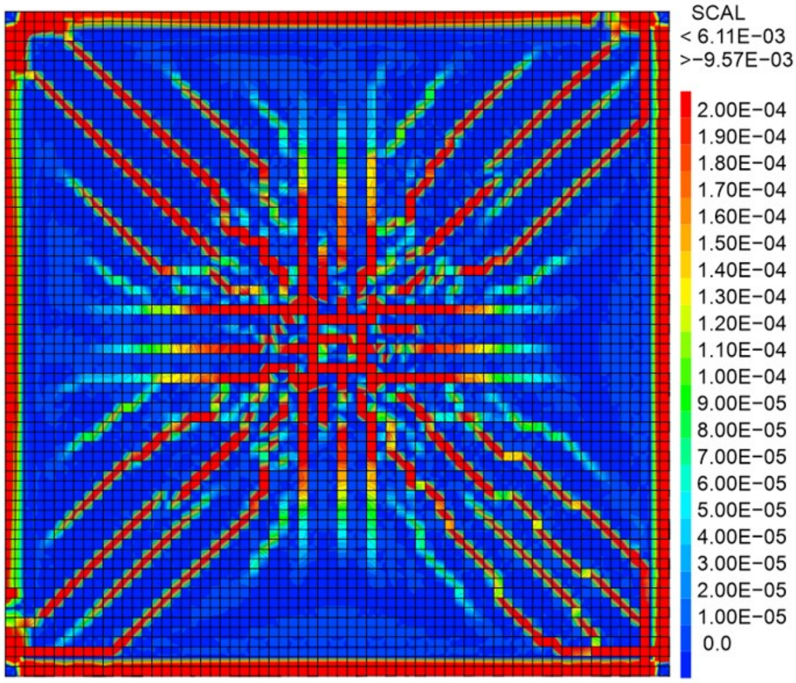
Fig. 8. Comparison of load–deflection curves obtained from the finite element model and experiment

2.2. Results

Fig. 8 illustrates the comparison between the load–deflection responses derived from the numerical model (FEM) and those obtained experimentally (EXP) [5]. The results reveal a good degree of agreement, with the simulated curve accurately reproducing the structural behavior observed in the experiment across both the elastic and nonlinear phases of the response. The ultimate load value shows no significant difference compared to the experimental results (Table 1), with an error of less than 4%.



Experiment results [5]



Simulation results

(a) SC slab

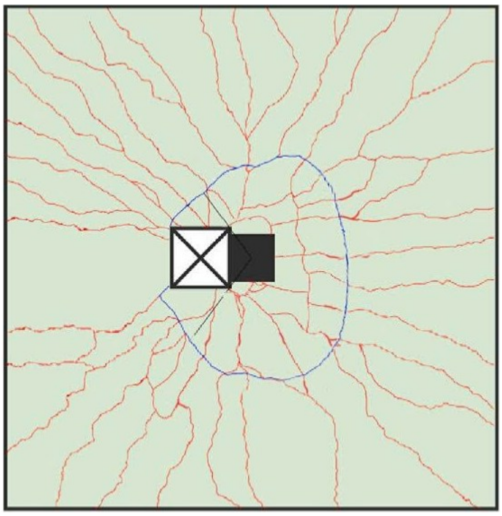
Fig. 9. Failure pattern of the slab in the experimental test [5] and numerical simulation

Fig. 9 presents the simulated failure patterns of the tested slab specimens. The simulated failure mode closely matches the experimental observations, with radial cracks appearing at the slab's bottom surface around the column perimeter and circumferential cracks forming a truncated cone in the control specimen. In contrast, the S-SO-200-0 specimen exhibited radial cracks originating at the corners of the adjacent square opening and propagating toward the slab edges, accompanied by asymmetrically distributed tangential cracks that formed an irregular punching cone, characterized by a reduced diameter on the opening side and an enlarged diameter on the opposite side.

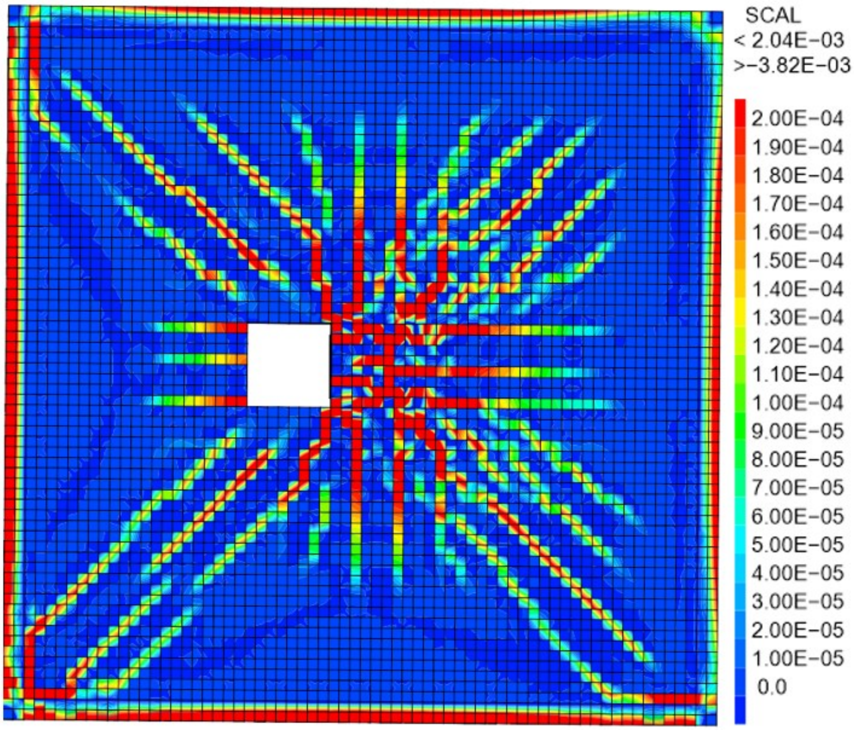
3. Parametric investigation on the influence of openings

The validated numerical model exhibited a high level of consistency with the experimental results in terms of both the load–displacement response and the observed slab failure modes. On this basis, the model was adopted as a reliable analytical tool for conducting a parametric investigation into the influence of location and size of openings, which were analyzed.

The opening sizes and locations in this study are selected based on practical construction requirements. The chosen opening dimensions are representative of typical requirements for mechanical, electrical, and plumbing systems. In practice, openings are often arranged near columns to facilitate the integration of service ducts and to satisfy architectural and functional requirements. From a structural perspective, openings located in this region coincide with the critical punching shear zone, where stress concentration is highest and the influence on punching behaviour is most pronounced.



Experiment results [5]



Simulation results

(b) S-SO-200-0

Fig. 9. (continued)

3.1. The effect of opening location.

This section investigates the influence of opening location on punching shear capacity. A 200 × 200 mm square opening was placed at distances of 0 mm, 50 mm, and 100 mm from the column, with material properties kept constant.

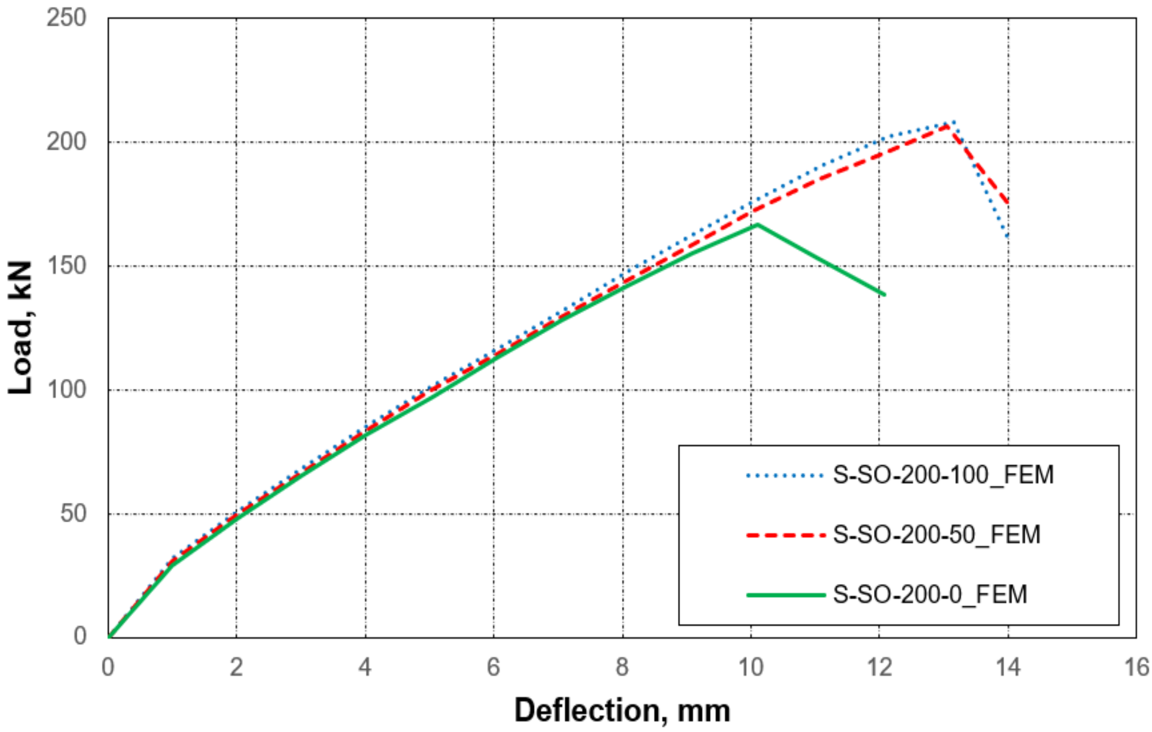
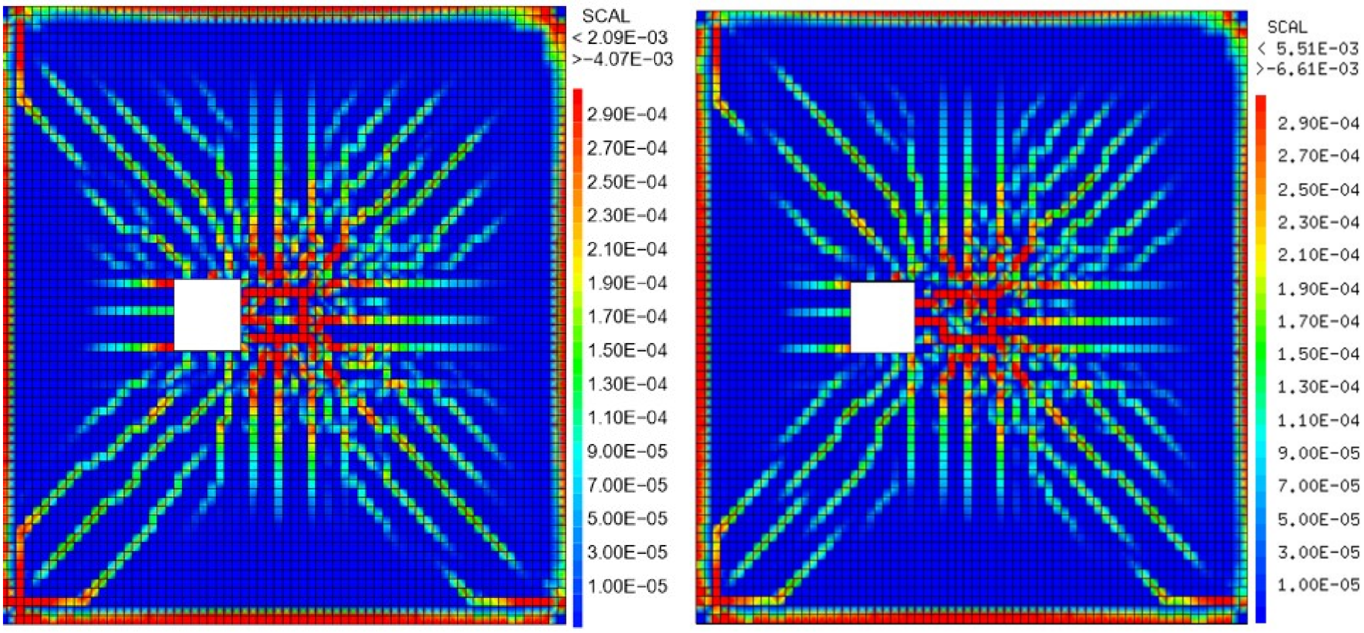


Fig. 10. Load–deflection curves for slabs with varying opening–column distances



(a) S-SO-200-50 (b) S-SO-200-100

Fig. 11. Failure patterns of slabs with different opening–column distances

As shown in Fig. 10, the slab with the opening adjacent to the column (S-SO-200-0) had the lowest stiffness and ultimate load at 166.73 kN, while moving the opening to 50 mm increased the ultimate load to about 206.5 kN. The best performance was observed at 100 mm (S-SO-200-100), reaching nearly 208.3 kN, close to the reference slab. Fig. 11 further shows that both specimens exhibit radial cracks propagating from the column region toward the slab edges, confirming punching shear as the governing failure mechanism. In specimen S-SO-200-50, cracks are denser and more concentrated at the column–opening interface, with multiple intersections indicating strong stress interaction and reduced stiffness. By contrast, specimen S-SO-200-100 shows more uniformly distributed cracks around the column and less concentration at the opening edge; secondary cracks are less interconnected with the main radial cracks, reflecting improved stability and enhanced punching shear capacity.

3.2. The influence of opening size

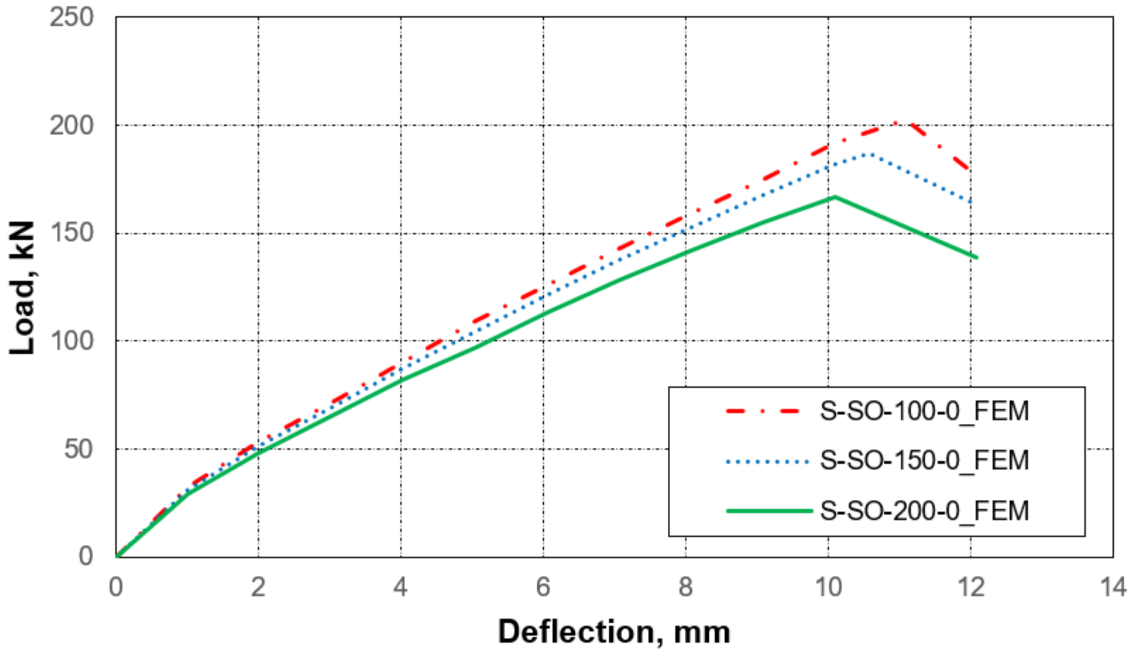
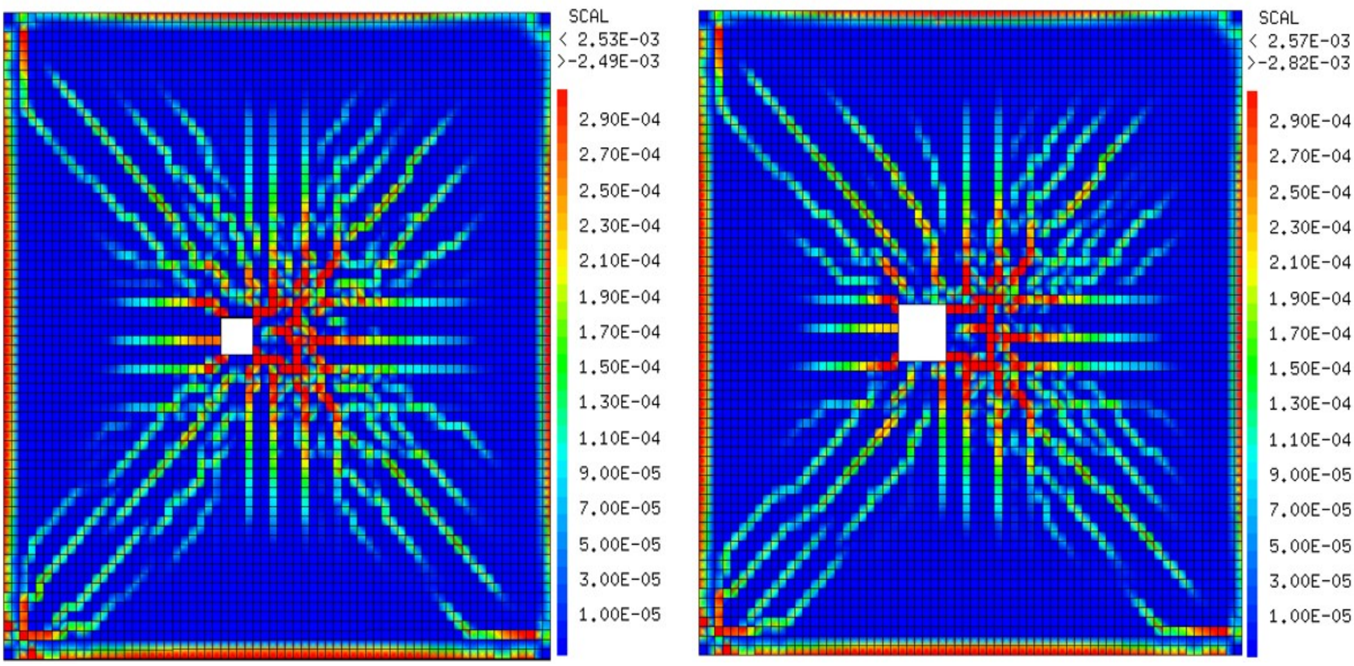


Fig. 12. Load–deflection curves of slabs with different opening sizes



(a) S-SO-100-0 (b) S-SO-150-0

Fig. 13. Failure patterns of slabs with different opening sizes

The openings with dimensions of 100 × 100 mm (Specimen S-SO-100-0) and 150 × 150 mm (Specimen S-SO-150-0) were selected to evaluate the influence of opening size on the punching shear behavior of the slab. The material properties of concrete and reinforcement, as well as the reinforcement ratio, were kept constant.

As shown by the load–deflection curves in Fig. 12, slab stiffness decreased with the opening size expanded from 100 to 200 mm, and punching shear capacity was reduced by 8.3% and 12.4% when enlarging the opening from 100 to 150 mm and from 150 mm to 200 mm, respectively. Fig. 13 illustrates that both specimens exhibit radial crack patterns propagating from the column. region and the edges of the openings toward the slab boundaries, indicating a punching shear failure mechanism. The primary cracks in the two slabs are largely similar in distribution and direction, suggesting that both opening sizes (100 × 100 mm and 150 × 150 mm) induce comparable failure modes.

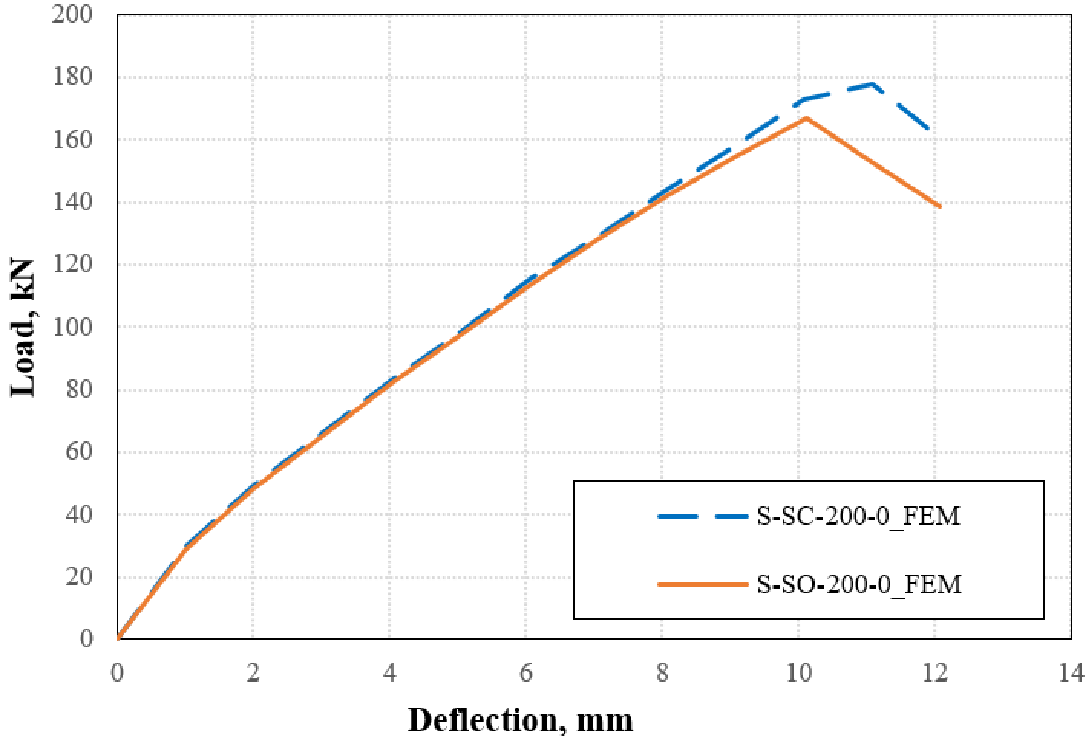


Fig. 14. Load–deflection curves of specimen S-SC-200-0

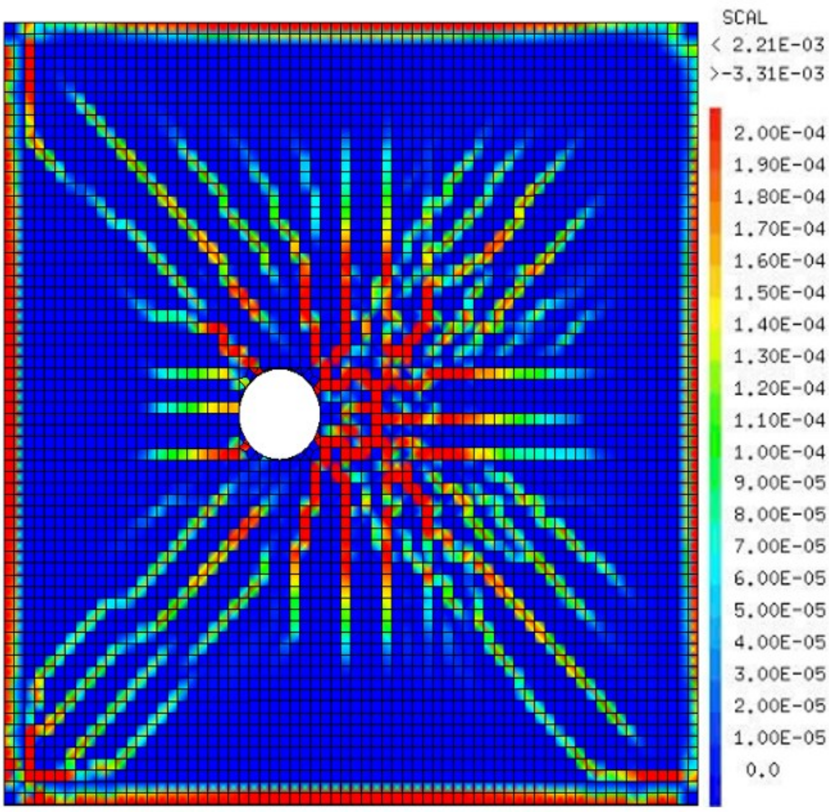


Fig. 15. Failure patterns of slabs with circular opening

3.3. The effect of opening shape

This section investigates the influence of opening shape on the punching shear behaviour of flat slabs. To isolate the effect of geometry, two specimens with identical opening size and material properties were considered: a circular opening with a diameter of 200 mm (S-SC-200-0) and a square opening with an equivalent characteristic dimension (S-SO-200-0), both positioned adjacent to the column.

As shown in Fig. 14, the specimen with a circular opening exhibits higher initial stiffness and a greater ultimate punching load (approximately 177.6 kN) compared to the specimen with a square opening (approximately 166.7 kN). This difference can be attributed to the reduced stress concentration associated with curved geometries. In contrast, the presence of sharp corners in square openings leads to localized stress amplification, which accelerates crack initiation and propagation in these regions.

The crack patterns presented in Fig. 15 further illustrates this behaviour. For the circular opening, cracks are distributed more uniformly around the column–opening interface and propagate radially outward, indicating a relatively smooth stress redistribution process. On the other hand, for the square opening, cracks tend to initiate and concentrate at the corners of the opening, where stress concentration is highest, resulting in earlier stiffness degradation and reduced load-carrying capacity.

These observations highlight the significant influence of opening shape on the punching shear response of flat slabs. Openings with rounded geometries are more effective in maintaining structural integrity by mitigating stress concentrations, whereas openings with sharp edges promote localized damage and premature failure. Therefore, from a structural design perspective, the use of circular or rounded openings is preferable when openings must be placed near slab–column connections.

4. Conclusion

The paper has presented a numerical investigation using the finite element method to evaluate the punching shear behavior of reinforced concrete flat slabs with openings. A close correlation was observed between the Mazars model predictions and the experimental results for crack patterns and load–deflection response, with an ultimate load capacity difference of less than 4%. On this basis, a parametric study was conducted to examine the effects of opening location and opening size. Several key conclusions can be drawn as follows:

Effect of opening location: Openings placed adjacent to the column significantly reduced punching shear strength. The ultimate load of the slab with a 200 × 200 mm opening at the column edge was 166.73 kN. Increasing the distance from the opening to the column to 50 mm and 100 mm resulted in ultimate loads of 206.5 kN and 208.3 kN, representing increases of 23.86% and 24.93%, respectively.

Effect of opening size: Enlarging the square opening from 100 × 100 mm to 150 × 150 mm and then to 200 × 200 mm caused successive reductions in punching shear resistance of 8.3% and 12.4%, respectively. Larger openings also led to lower initial stiffness, as reflected in the load–deflection curves.

Effect of opening shape: The slab with a circular opening achieved an ultimate load of 177.6 kN, higher than the slab with a square opening of the same size (166.7 kN). The rounded geometry of circular openings effectively reduced stress concentrations at corners, resulting in improved stiffness and punching shear resistance.

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