Journal of Science and Transport Technology
Journal homepage: https://jstt.vn/index.php/en

KD-Railway 1.0 – A structural dynamics software for high-speed rail bridge based on open source Cast3m platform

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Article info
Type of article:
Original research paper

DOI:
https://doi.org/10.58845/jstt.utt.2022.en.2.2.39-48

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Received: 06/04/2022
Revised: 14/06/2022
Accepted: 17/06/2022

Abstract: Creating calculation tools, algorithms, element models, and non-linear materials based on open sources has recently received the attention of scientists and companies. The paper generalizes one practical tool for analyzing the structural dynamics for the high-speed railway bridge based on Cast3M open-source. Compared with the classical approach with the analytical formulation, KD-Railway developed by the finite element method allows defining the structure with complex geometry and considering a different type of structure (truss, cable, hybrid composite) or the structure-soil interaction. Compared with the commercial software, KD-Railway meets two critical objectives for practical applications: minimizing the number of input parameters and fast calculation time due to the integration of sharing resource technology and parallel calculation of Cast3M. This paper clarifies the theoretical background and critical functionalities of KD-Railway. The validation process used 4 case studies covering the different moving load models, including the only moving load, a series of moving loads, and the real moving load model representing the conventional trains, applied for the simply-supported beam and continuous beam bridges. All the code of this first version of KD-Railway is available online.

Keywords: High-speed railway bridge, Open source, Finite Element Analysis (FEA), Cast3M, KD-Raiway

1. Introduction

The dynamic response of the interaction between train, track, and bridge under high-speed train's movement affects both the vehicles and the structures in a complex manner. With a traffic speed of over 200 km/h, the resonance effect due to the repetition of the axle load can cause harmful consequences for the bridge, such as the ballast base instability, discomfort to passengers due to the vertical acceleration, and an increase in rail maintenance costs [1]–[3]. In modern rail bridge design standards, vertical displacement and acceleration are two of the most stringent specifications that must be scrutinized.

The moving constant force model is the simplest and earliest model widely employed in researching the dynamic behavior of the high-speed bridge [4]–[6]. Then the moving harmonic force model was proposed in the early twentieth century. The eccentric forces of locomotives are considered as moving harmonic forces investigated the resonance problem of bridge structures [7]. This approach does not consider the dynamic interaction between train and bridge,

Published online 28/06/2022

leading to the fact that the moving force model is only useful for the cases the weight of the train is much smaller than that of the bridge, and the dynamic behavior of the train is not of interest. When the mass of the vehicle cannot be ignored, the moving mass model should be utilized instead by considering the mass and inertia of the running vehicle [5]. On this basis, the train was simulated by the moving spring-damping-mass system in which the suspension system is simplified to a moving mass supported by a spring-damping element [8], [9].

After the 1960s, with high-performance computers and the Finite Element Method (FEM) development, the train–bridge dynamic interaction model was investigated. In this modern model, the theory of multi-body system dynamics is adopted to simulate the train subsystem, while the bridge subsystem is usually modeled based on FEM [10]–[14]. The dynamic response of high-speed railway bridges was recently investigated using the machine learning model [15]. This idea tends to improve the effectiveness of the simulation design, but the model's performance still depends on the database generated by FEM method.

Among these numerical approaches summarized above, the Finite Element Method implemented in the commercial software for structural design such as Midas, Sap2000, Robot Structural Analysis, or specialized software for structural simulation such as Ansys, Abaqus for modeling the bridge system has gained large importance. They simulate and calculate complex systems with highly accurate results. However, the model creation procedure has the main drawback of the time-consuming with the high number of dynamic analyses required. Significantly, the definition of the dynamic load step is backbreaking work. It demands creating many load functions, each corresponding to a different position and time of train movement when crossing the bridge. There is no commercial software that allows automation of the moving load definition function in the authors' knowledge. To solve the problem, the engineer needs to use external interaction codes in the form of Application Programming Interface (API) developed separately for each software.

The commercial software has a disadvantage in the cost of copyright and requires a high machine configuration for the modeling mode calculated through the user interface. That is one of the reasons the open-source for structural analysis has been developed in the last two decades, thanks to their open access, free education, and research. Open-source promotes a free exchange of ideas within a community to drive creative, scientific, and technological advancement to remove barriers between innovators. In addition, the software developed using an open-source solver can be compiled and run on different operating systems, allowing for the maximum distribution and use of the machine's resources.

Cast3M [16] is a finite element analysis (FEA) open-source applied in solid and fluid mechanics. Cast3M integrates a library of many modeling functions, allowing users to create a complex structure for dynamic and non-linear structural analysis [17]–[20]. However, Cast3M in particular and other open-source for simulation, in general, are not easy to use. When working with an open-source code, you need to do significant work to be familiar with and use the code for the first simulation project.

Based on the above reasons, KD-Railway was developed to overcome the disadvantages of using commercial simulation software and open source FEA. This software uses code Cast3M for the solver and integrates the interface developed in C# and Python, for dynamic structural analysis of high-speed railway. This paper will present the basic algorithms for railway dynamics and the primary functionalities of KD-Railway version 01 with the validating results compared with different analytical methods and experimental measurements that existed in the literature.

2. Theoretical Background & Functionalities of KD-Railway

KD-Railway is three-dimensional for determining the response of the track structure under static and dynamic loading using the finite element method-based code Cast3M. The first version focuses on calculating the dynamic response of the high-speed rail bridge. We use the time history modal superposition dynamic analysis (MSDA) to overcome the disadvantages of the commercial simulation software in time-consuming for both simulation and calculation steps. The equation of dynamic motion of the bridge can be expressed as:

$$[M_b]\{\ddot{V_b}\} + [C_b]\{\dot{V_b}\} + [K_b]\{V_b\} = \{P_b\} \tag{1}$$

In the Modal Superposition Method, only the first N_0 modes of the bridge are contributing to the interaction computation, and the modal data are normalized to the mass matrix:

$$\Phi^T[M_b]\Phi = 1 \tag{2}$$

Thus, the mass matrix of the bridge subsystem in modal coordinates is an N_0-order unit matrix, the damping and stiffness matrices of the bridge subsystem are diagonal matrices:

$$[M_b^*] = \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}$$

$$[C_b^*] = \begin{bmatrix} 2\varepsilon_1\omega_1 & & & & \\ & 2\varepsilon_2\omega_2 & & & \\ & & 2\varepsilon_3\omega_3 & & \\ & & & \ddots & \\ & & & & 2\varepsilon_{N0}\omega_{N0} \end{bmatrix} \tag{3}$$

$$[K_b^*] = \begin{bmatrix} \omega_1^2 & & & & \\ & \omega_2^2 & & & \\ & & \omega_3^2 & & \\ & & & \ddots & \\ & & & & \omega_{N0}^2 \end{bmatrix}$$

Where ω_i and ε_i are the frequency and the damping ratio of the ith mode. The motion equations of the bridge can be expressed as:

$$\ddot{q} + C_b^*\dot{q} + K_b^*q = P_b^* \tag{4}$$

where q represents the displacement in modal coordinates, P_b^* is the normalized force.

As stated in the introduction, the definition of time-load functions for the moving load is complicated because both factors, time and loading force location, are changed during train movement. For example, a train with 250m long, passing over a 40m long bridge, the load functions are defined in each 0.5m position steps, so the user needs to define about (250+40)/0.5=580 different load functions for one speed. If we calculate for the speed range from 100km/h to 350km/h with a step of 5km/h, for 10 trains of the HSLMA group according to European standards, the total load function to be defined will be 580 x 51 x 10 = 295 800 functions. When using commercial software such as Abaqus, Ansys, SAP2000, etc . the engineer needs to create API macros to create these load functions. This approach is challenging, causes confusion errors, and consumes the engineer's time in the model building step.

Assessing the difficulties and shortcomings related to the use of commercial software to calculate the dynamics of the HSR bridge above, the KD-Railway software was built with two main goals: to simplify the process of simulating and reduce computation time. The interface was designed to be compact and minimize the number of input parameters, helping users save time in model building and avoid unnecessary errors.

KD-Railway's user interface is built in C# (Figure 1). The main functionalities of the first version are below:

- Dynamic analysis of high-speed railway bridges by the finite element method, using modal superposition dynamic analysis method

- Automatically build load functions over time, shortening simulation and calculation time compared to commercial software such as Midas, Sap2000

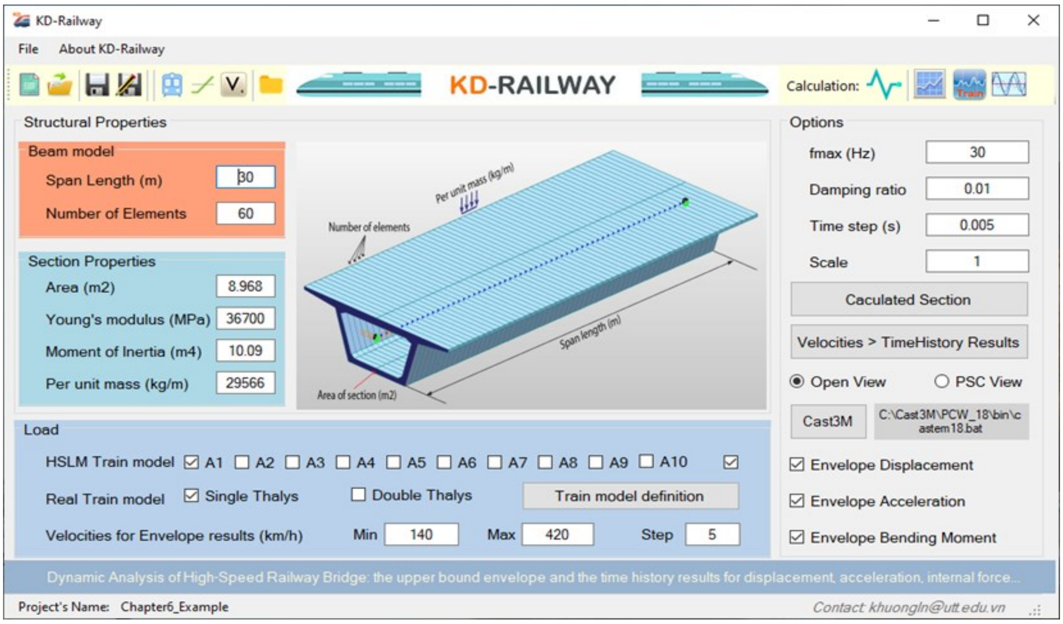
- Quickly draws the envelope diagrams in velocity and the time history graphs of displacement, acceleration, and internal force.

- Integrated Eurocodes standard load models and supported the creation of reports with vector results

Another difficulty in calculating the dynamics response of high-speed rail bridges under moving loads is the calculation time. Even if we choose the MSDA approach, the calculation still faces many challenges because the time step and the number of modal modes also affect the calculation results. In addition, if the user does not select the appropriate result processing, it will cause an overflow of the computer's internal memory due to many computations and the results.

An example of optimization of computation time with Abaqus software has been clarified in the research of Mellier [21]. The author combined MATLAB software to create a computational model on Abaqus in that study. Regardless of the time to build the calculation model and define the load functions as analyzed above, if only considering the calculation time directly on Abaqus software, the author needed 134 minutes, 40 minutes, and at least 22 minutes for 03 options input parameter selection and output processing. In other words, even if the author automated the entire step of creating the model and selecting the input parameters, the dynamic analysis still requires 22 calculation minutes for ten moving loads (HSLMA1-10) traveling at different speeds in the speed range from 100km/h to 350km/h.

Regarding computation time, KD-Railway integrates and exploits Cast3M's parallel computing and resource threading technology to improve computational efficiency. In a specific case, KD-Railway resolved to calculate the dynamics response of a 32m-span girder bridge under 10 load models of HSLMA moving with a speed range from 150km/h to 450km/h problem in 16 seconds of calculation (Figure 2), this time calculation is considerably shorter than 22 minutes optimized process effected in the research of Mellier [21].



Structural Properties

Beam model
Span Length (m) 30
Number of Elements 60

Section Properties
Area (m2) 8.968
Young's modulus (MPa) 36700
Moment of Inertia (m4) 10.09
Per unit mass (kg/m) 29566

Options
fmax (Hz) 30
Damping ratio 0.01
Time step (s) 0.005
Scale 1
● Open View ○ PSC View
C:\Cast3M\PCW_18\bin\castem18.bat

Load
HSLM Train model ☑ A1 ☐ A2 ☐ A3 ☐ A4 ☐ A5 ☐ A6 ☐ A7 ☐ A8 ☐ A9 ☐ A10 ☑
Real Train model ☑ Single Thalys ☐ Double Thalys
Velocities for Envelope results (km/h) Min 140 Max 420 Step 5

☑ Envelope Displacement
☑ Envelope Acceleration
☑ Envelope Bending Moment

Dynamic Analysis of High-Speed Railway Bridge: the upper bound envelope and the time history results for displacement, acceleration, internal force...

Project's Name: Chapter6_Example

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Figure 1. KD-Railway version 1.0 user interface

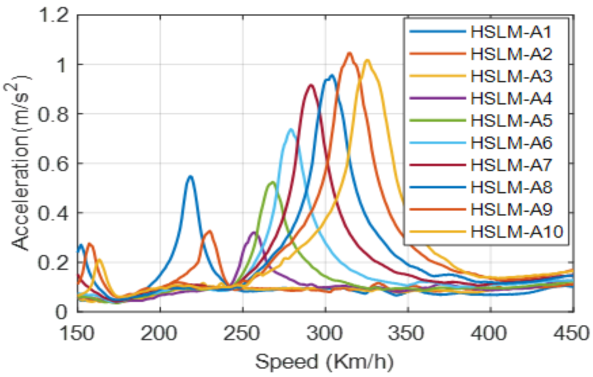


Figure 2. Acceleration envelope curves for HSLMA system

3. Case studies for validation

In this section, 04 case studies will be described and used to validate the accuracy of KD-Railway. These case studies cover the different types of moving load models, including the only one moving load, a series of moving loads, and the real moving load model representing the conventional trains. The results in case 01 and case 02 were determined from the analytical methods, while the results in the third and fourth samples were measured from the experimental test. These case studies are also helpful for the future development of high-speed railway bridge tools where the users can use the results to validate the proposed methods.

Only structural properties related to the material and geometries are described herein. The main results of each case study need to be consulted from the original references.

3.1. Problem 01: A simply-supported beam subjected to a moving load

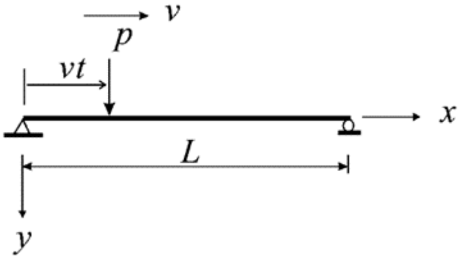


Figure 3. Beam subjected to a moving load

A simple beam with L=20m (Figure 3) is subjected to a load of magnitude p = 6kN moving at speed v=100 km/h. The mass per unit length m=3000kg/m, the modulus of elasticity E, and the moment of inertia I of the beam give EI=109N/m. For this structure, the vertical deflection and vertical acceleration of the beam (along the y axis) at position x and time t with different moving speeds were calculated. The analytical method was proposed by Yand et al. [22].

3.2. Problem 02: Simple Beam Subjected to a Series of Moving Loads

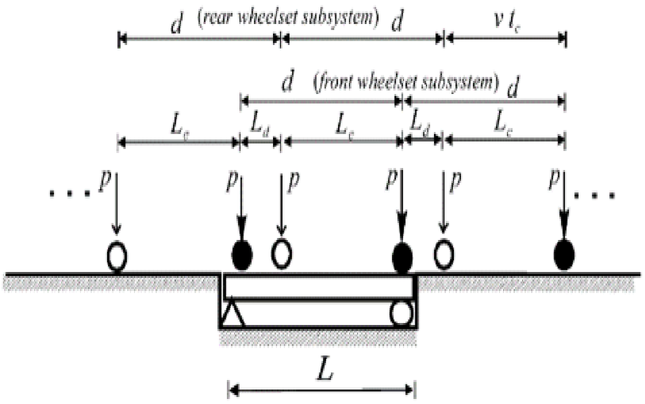


Figure 4. The simple beam subjected to train loads

Similar to the first problem, a simply-supported beam bridge was considered. The main structural and material properties were assumed, including the length L = 20m, the moment of inertia I = 3.81m4, the modulus of elasticity E = 29.43 GPa. Unlike case 01, in this problem, a series of moving loads representing a train with N = 5 cars of identical length d = 24m. The car's two-wheel assemblies (or bogies) are separated by 18 m, i.e., L_c = 18m and L_d = 6m (Figure 4). The mass of each wheel assembly is M = 22000 kg, corresponding to p = 215.6 kN.

Yang et al. [22] proposed an analytical method to determine the deflection along the y axis at the midpoint of the bridge at two speeds: v=34m/s, which is indicative of the resonance phenomenon, and v=26m/s which is indicative of the cancellation phenomenon. The midpoint deflection of the beam was calculated and represented in the form of transilient results.

3.3. Problem 03: Dynamic analysis of a high-speed railway bridge under Thalys trains

This problem considers a bridge composed of multi-span simply supported PC girders with spans of 50m and U-shaped sections. The cross-section properties of the bridge were calculated from the original dimensions highlighted in the paper of Xia et al. [23]: the moment of inertia I = 56.48m^4, the section area A=25.57 m^2, the mass per unit length m=77925 kg/m. The modulus of elasticity of the material is E = 29.43 GPa.

For this case study, several experimental tests were conducted to measure the deflection and acceleration of the bridge under the high-speed Thalys trains with articulated vehicles. The speeds of the Thalys trains were between 265 and 310 km/h. The measurements were the form of time-history results.

3.4. Problem 04: The continuous railway bridge crosses the river Viskan

A continuous bridge with two spans and a total theoretic length of 45 m was considered for this case study. Theis railway bridge crosses the river Viskan and is located between the cities Gothenburg and Varberg (km 63 + 145).

The properties of the bridge were calculated from the literature [24], including the total mass per unit length of the bridge 19000 (kg/m); the moment of inertia I = 1.16 m^4. In this research. Modulus of elasticity of the material used for this structure is E = 3.4e10 (Pa).

In the research conducted by Lena [24], the two program, LUSAS and DYNSOLVE, were used to analyze the dynamic response of the bridge subjected to the HSLM-A trains, inclusive A1 to A10.

4. Validation

In this section, the results obtained by KD-Railway will be presented and compared with those existing in the literature. The maximal displacement and acceleration that appear during the train passing over the bridge will be captured to plot the envelope of displacement and acceleration versus the moving speed. The primary purpose of these studies is to demonstrate the accuracy of the KD-Railway's calculation and its performance in terms of computation time and accuracy.

4.1. Case study 01

The displacements for the midpoint of the beam obtained by KD-Railway are illustrated in Figure 5 and Figure 6. The results for the damped and undamped cases have been compared. As can be seen, the effect of damping on the response of the beam during the action of the moving load is relatively small. This phenomenon was also highlighted in the original research by Yang et al. [22].

Similarly, the distribution curves of bridge deflection versus moving load speed considering only the contribution of either multi-modes or the first mode have been plotted in Figure 6. The non-dimensional speed parameter S_1 defined as the ratio of the frequency of excitation of the moving load to the first frequency of vibration of the beam, was used in the analytical results of Yang [22].

$$S_1 = \frac{\pi v}{\omega_1 L} \qquad (5)$$

As is shown, the midpoint displacement impact response in this case of the simple beam subjected to one moving load is dominated by the first mode, and both methods well predicted the results.

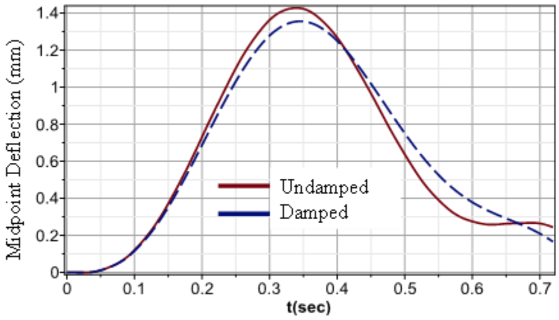


Figure 5. Vertical displacement at the midpoint of the beam in case of speed v=100km/h

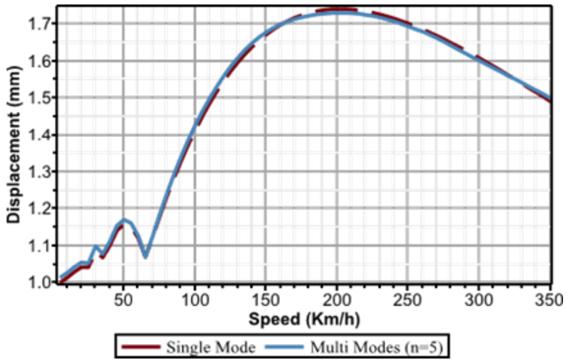


Figure 6. The maximum vertical displacement at the midpoint in a speed range of 0km/h-350km/h

4.2. Case study 02

The midpoint responses of the beam subjected to the action of the wheel loads moving at the two speeds of 34 m/s (122.4 km/h) and 26 m/s (93.6 km/h) have been plotted in Figure 7 for results obtained from KD-Railway program.

As can be seen, for the two cases of velocity considered, the analytical solution proposed by Yang et al. [22], which was obtained by considering multi mode of vibration of the beam, agrees very well with the finite element result. The resonance phenomenon and a typical cancellation phenomenon related to two trains' speeds of 34 m/s and 26 m/s are well predicted by KD-Railway.

For the same load model, the maximum displacement computed for the midpoint of the beam has been calculated and plotted against the speed in Figure 8. First, it can be noted that the KD-

Railway result agrees very well with the analytical solution in Yang *et al.* [22]. Second, the resonance points with S1 = 0.60, 0.15, 0.12 corresponding to speeds v = 600, 150, 120 km/h, respectively, can generally be observed. In contrast, the other resonance points are merely suppressed as they are coincident with or close to the points of cancellation.

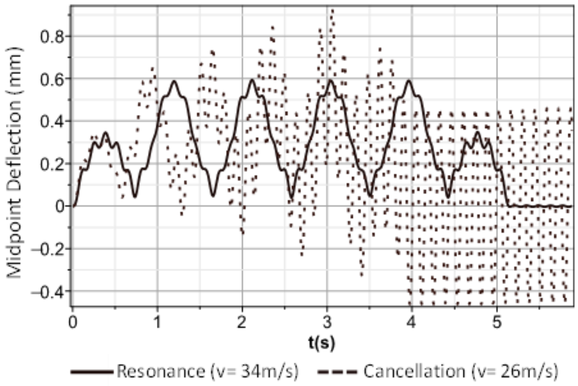


Figure 7. Vertical displacement at the midpoint in two speeds: 34m/s and v=26m/s

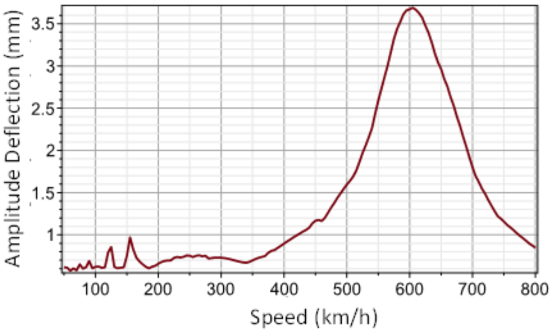


Figure 8. The maximum vertical displacement at the midpoint in a speed range of 100km/h-800km/h

4.3. Case study 03

The vertical deflection obtained by KD-Railway and measured by the LVDT on the stiff antenna at the center of the span of the girder under single Thalys and double Thalys trains were compared. The displacements calculated for 2 types of Thalys train model by KD-Railway are shown in Figure 9 and Figure 10. The maximal deflections of the bridge under the single Thalys and the double Thalys running on the passage side of the measurement were 1.6 and 1.75 mm, respectively, and KD-Railway predicts these values with the numerical displacement 1.62 and 1.75 mm. It can be noted that the displacement-time curve calculated by KD-Railway is very close to the experimental measurement results presented in the research of Xia et al. [23].

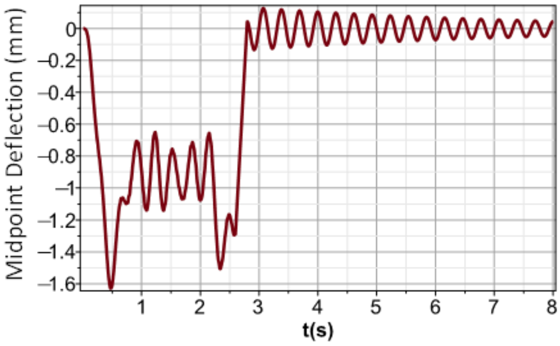


Figure 9. Vertical displacement at the midpoint for single Thalys train moving at v=295 km/h

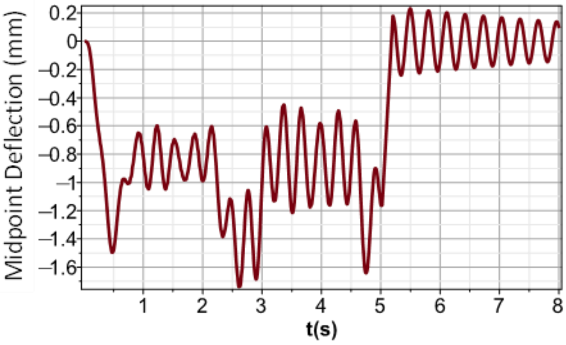


Figure 10. Vertical displacement at the midpoint for double Thalys train moving at v=295 km/h

The typical vertical acceleration curves under double Thalys trains obtained by KD-Railway is shown in Figure 11. It could see that there is no obvious difference between the vertical accelerations induced by KD-Railway and measurement obtained by Xia et al. [23]. The maximum acceleration at the mid-span is about 0.5 m/s^2 for both methods.

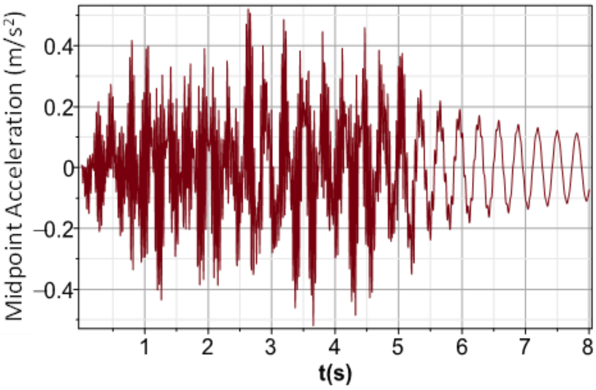


Figure 11. Vertical displacement at the midpoint for double Thalys train moving at v=295 km/h

The envelope curves of vertical acceleration of the bridge under 6 common real train models are presented in Figure 12. These train models were described in ERRI report [25] and defined by the user's load definition option of KD-Railway. This functionality is very helpful for the new calculation with different train load models.

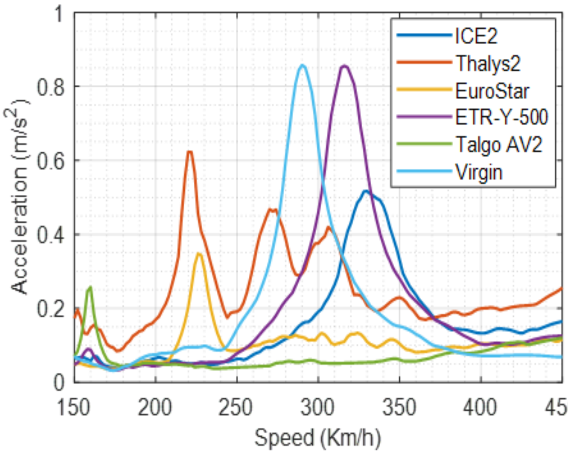


Figure 12. Acceleration envelope curves for real trains

In this case study, the results obtained by KD-Railway were compared with the experimental measurements, proving the reliability and applicability of the software in practice. It is important to note that the numerical results exploited in this study were extracted from the point on the section's neutral axis. We need to incorporate additional techniques such as multi-fiber beam elements or rigid crossbars to determine the acceleration at different measuring points on the cross-section. These functionalities will be implemented in the next versions of KD-Railway.

4.4. Case study 04

The structure is a continuous bridge over two spans and has a total theoretic length of 45 m. Maximal vertical accelerations versus the 10 conventional HSLM-A train's speeds range of 70-370 km/h are presented in **Figure 13**. The results calculated with a damping ratio of 1%, are very close to those obtained by 02 different dynamic analysis programs, including LUSAS and DYNSOLVE conducted by Lena [24]. From the comparison, it can be noted that KD-Raiwlay is able to predict the dynamic response, resonance, and cancellation phenomenon. The results demonstrate the simulation ability and reliability of KD-Railway in dynamic analysis of continuous girder bridges. In addition, the efficiency of simulation and calculation time of KD-Railway is also shown in this case. The total time consuming for getting out the results presented in Figure 13 is 18 seconds. In comparison, the time calculated by two software used by Lena [24] was nearly 20 minutes.

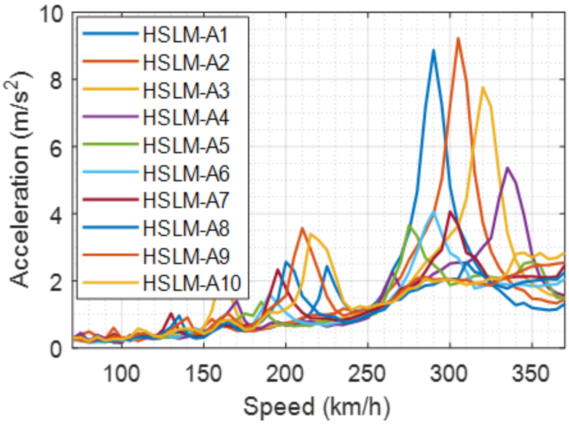
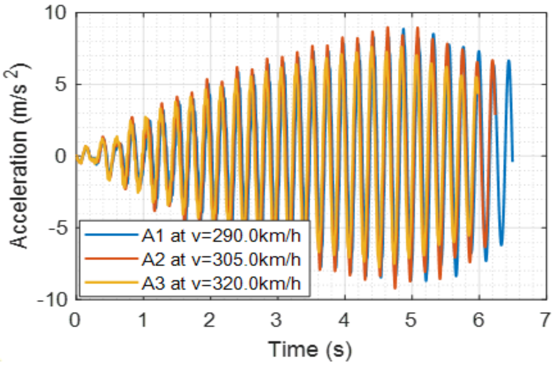
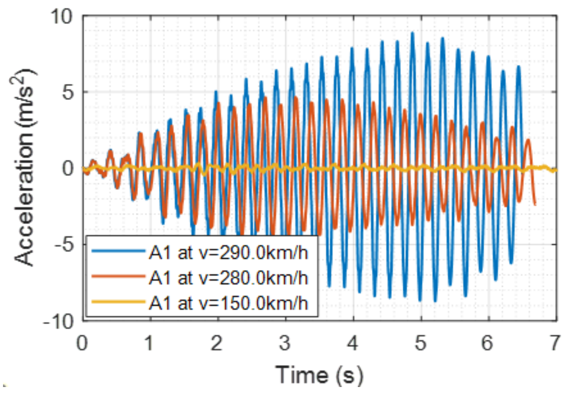


Figure 13. Maximum vertical acceleration of the bridge versus a speed range of 70km/h-370km/h

Figure 14(a) presents the accelerations in the form of time history curves at the middle section of the bridge for 3 train models, HSLM-A1, HSLM-A2, HSLM-A3, at the resonance speeds of 290km/h, 305km/h, and 320 km/h, respectively. Similarly, Figure 14(b) presents the acceleration against time for HSLM A1 train model at three speeds of 290km/h, 280km/h, and 150km/h. The resonance phenomenon is well described in these results, with only a variation of 10km/h in speed; the maximal acceleration varies two times.



(a)



(b)

Figure 14. Time history of acceleration at different speeds

5. Conclusion

Creating calculation tools, algorithms, element models, and materials based on open-source has recently received the attention of scientists and companies. The paper generalizes one practical open-source application in structural simulation using the finite element method. The theoretical background and critical functionalities of KD-Railway software are clarified and verified by comparison with the literature's analytical method and measurement results. The study focused on sharing the process of developing a specialized computational tool for analyzing railway bridge dynamics based on open-source code Cast3M and the case studies for validating these types of structures. All the code of this first version of KD-Railway is available online: https://github.com/lekhuong/KD-Railway

In comparison with the classical method, where the research proposed the close analytical form for the dynamic analysis, KD-Railway developed by the finite element method allows defining the structure with complex geometry such as the variable cross-section, multi-span with continuous beam bridge, taking into account the structure-soil interaction.

In addition, KD-Railway meets two critical objectives for practical applications: minimizing the number of input parameters and fast calculation time due to the integration of sharing resource technology and parallel calculation of Cast3M. For the first version, KD-Railway allows users to:

- Analyze the dynamic response of simple and continuous beam bridge;

- Automatically build load functions over time;

- Shortened simulation and calculation time compared to commercial software such as Midas, Sap2000;

- Integrate Eurocodes standard load models;

- Integrate different train models in standard design, including the real train model;

- Quickly draws the envelope diagrams in velocity and the time history graphs of displacement, acceleration, and internal force;

- Create the reports with vector results.

The next version of KD-Railway will include new functions such as dynamic calculation for truss bridges and cable-stayed bridges, using different elements like multi-fiber beams or shell elements to cover the bridge dominated by the torsion modes.

Acknowledgments

This research is funded by University of Transport Technology (UTT) under grant number ĐTTĐ2021-28.

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