



Experimental evaluation of modulus of elasticity of concrete using limestone and basalt aggregates determined according to different standard test methods

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Abstract: The modulus of elasticity of concrete is influenced not only by its compressive strength but also by the geological origin of the coarse aggregate used. However, current standards employ diverse testing methods and predictive equations, many of which neglect the aggregate's contribution. This study presents experimental data for concrete with compressive strengths ranging from 30 MPa to 55 MPa, produced using two Vietnamese coarse aggregates: Ha Nam limestone and Hoa Binh basalt. The modulus of elasticity was measured according to three standards: EN 12390-13:2021, ASTM C469-10, and TCVN 5726-2022. At a constant water-to-cement ratio, concrete incorporating Ha Nam limestone exhibited comparable compressive strength to that of Hoa Binh basalt, yet demonstrated a higher elastic modulus. Based on the experimental data, this study proposes using preliminary coefficients of 21.000 and 19.700 - corresponding to Ha Nam limestone and Hoa Binh basalt aggregates, respectively - for estimating the modulus of elasticity in accordance with EN 1992-1-1:2004. For estimations in accordance with ACI CODE-318-25, the recommended preliminary factors are 5.250 and 4.810, respectively.

Keywords: Concrete, Modulus of elasticity, Limestone, Basalt, Standard test method.

1. Introduction

Modulus of elasticity is a fundamental parameter used in the linear analysis of concrete, reinforced, and prestressed concrete structures. In composite materials such as concrete, this property is influenced by the proportions and elastic characteristics of the aggregate, the cement paste, and the bond between them. Among these factors, the type and properties of the aggregate exert the most significant influence [1-3].

Alexander M.G et al. [4] demonstrated that the modulus of elasticity of concrete is strongly affected by both the aggregate and cement types, with the age of the concrete also playing a critical role. Their findings indicated that andesite and dolomite aggregates produce significantly stiffer concretes compared to granite or quartzite.

In a study examining two commonly used aggregate types in South Africa, Beushausen H. et al. [5] concluded that concrete made with granite

aggregate achieved higher compressive strength but exhibited a lower modulus of elasticity than concrete made with andesite aggregate. Their research further revealed that predictive models accurately estimated the modulus of elasticity for concrete containing andesite aggregate, but were less reliable for concrete with granite aggregate. Specifically, the Eurocode model provided accurate predictions for concrete with andesite aggregate and for granite-based concrete within the 30–60 MPa compressive strength range. However, for high-strength concrete (60–120 MPa), the model significantly overestimated the modulus of elasticity, prompting the authors to propose a 30% reduction factor for Eurocode predictions for concrete incorporating granite aggregate.

Santos A. et al. [6] reported that the influence of Brazilian basalt and dolomite coarse aggregates on the modulus of elasticity was negligible when compared to the concrete's compressive strength class. Their study found that estimates based on the American standard were, on average, 24% lower than experimental values, while Eurocode predictions were approximately 13 % higher. These discrepancies may be attributed to the study's focus on concrete with compressive strengths between 20 MPa and 40 MPa.

Beshr et al. [7] investigated the modulus of elasticity of concrete incorporating four types of coarse aggregates. They concluded that the aggregate type has a more pronounced effect on the modulus of elasticity than on compressive strength. Their findings suggest that weaker aggregates tend to produce more ductile concrete compared to stronger aggregates. The static modulus of elasticity was higher in concretes containing steel slag and quartzitic limestone aggregates than in those prepared with calcareous and dolomitic limestone aggregates.

Gora J et al. [8] examined concrete mixtures using ten types of coarse aggregates sourced from Polish and Ukrainian igneous and sedimentary rocks, with water-to-cement ratios ranging from

0.70 to 0.28. Their study demonstrated a significant influence of aggregate origin on the modulus of elasticity. Moreover, the results revealed that the discrepancy between the measured modulus of elasticity and the values estimated based on compressive strength classes according to Eurocode reached up to 20 % for concretes containing basalt and granite aggregates.

A recent study conducted in Colombia [9] investigated both normal-strength concrete (up to 50 MPa) and high-strength concrete (ranging from 55 MPa to 95 MPa), using coarse aggregates of varied mineralogical origins, including diorite, granite, peridotite, basalt, chert, and marble. The results showed that the ratio of chord modulus of elasticity to the square root of compressive strength for high-strength concrete was only 76 % of that observed in normal-strength concrete.

Collectively, these findings underscore that the modulus of elasticity of concrete is influenced by multiple factors, including aggregate type, compressive strength, and testing methodology. Existing estimation models often yield inconsistent results due to variations in aggregate characteristics. Consequently, many countries have prioritized the development of tailored predictive models and empirical coefficients that reflect the properties of locally available aggregates.

Several studies using Vietnamese raw materials [10-11] have also highlighted the impact of material composition and compressive strength on the modulus of elasticity of concrete. However, these studies did not examine in detail the specific influence of aggregate type on this property.

Under static loading conditions, concrete exhibits a nonlinear stress–strain relationship. Initially, it behaves elastically; however, as the applied stress increases, microcracks initiate, leading to deviation from linearity and ultimately resulting in failure. The ratio of stress to strain within the elastic region defines the modulus of elasticity.

Different standard test methods prescribe

varying criteria for identifying the elastic deformation range of concrete. Accordingly, the secant modulus, chord modulus, and initial modulus of elasticity are determined based on EN 12390-13:2021 [12], ASTM C469-10 [13], and TCVN 5726:2022 [14], respectively. These values should be applied in accordance with relevant structural design standards.

Numerous experimental studies have investigated the influence of various factors on the modulus of elasticity of concrete, leading to the development of several predictive equations. Among these, equations (1) and (2) are the most widely used general formulations. Key influencing parameters typically include compressive strength, concrete density, and the characteristics of the constituent materials.

$$E = \alpha \times \sigma_B^b \times \gamma^c \quad (1)$$

where: E is the modulus of elasticity of concrete, MPa; σ_B is the compressive strength of concrete, MPa; γ is the density of concrete, kg/m³; α , b , and c are empirical coefficients.

The empirical coefficients in equation (1) exhibit considerable variation across studies [4], [6], [15]. These coefficients are typically derived from experimental data and influenced by the specific characteristics of the constituent materials and the methodology used to determine the modulus of elasticity. As a result, their values are often tailored to particular conditions and are explicitly defined within relevant design standards.

The European standard EN 1992-1-1:2004 [16] specifies equation (2) to estimate the secant modulus of elasticity E_{cm} , for concretes with quartzite aggregates:

$$E_{cm} = 22000 \times (f_{cm} / 10)^{1/3} \quad (2)$$

where: f_{cm} is the mean value of the concrete cylinder compressive strength, MPa.

For limestone and sandstone aggregates, the estimated value from equation (2) should be reduced by 10% and 30% respectively. For basalt aggregates, the value should be increased by 20%.

The American standard ACI CODE-318-25 [17] takes into account the characteristics of the aggregate through the unit weight of concrete (ranging from 1440 kg/m³ to 2560 kg/m³) when estimating the chord modulus of elasticity E_c , in MPa, using equation (3).

$$E_c = 0.043 \times w_c^{1.5} \times \sqrt{f'_c} \quad (3)$$

where: w_c is the density, unit weight, of concrete, kg/m³; f'_c is the specified compressive strength of concrete, MPa.

For normal-weight concrete (unit weight is about 2320 kg/m³), ACI CODE-318-25 provides a simplified equation (4).

$$E_c = 4700 \times \sqrt{f'_c} \quad (4)$$

The Vietnamese national standard TCVN 5574:2018 [18], based on Russian standard SP 63.13330.2012 [19], does not propose a specific equation for calculating the modulus of elasticity. Instead, it provides tabulated values for the initial modulus of elasticity corresponding to various compressive strength classes of concrete. These tabulated values can be approximated using a regression equation (5) that yields a coefficient of determination R^2 equal 0.994, indicating a high level of correlation.

$$E_b = 10,695 \times \ln(B) - 4,55 \quad (5)$$

where: E_b is the initial modulus of elasticity of concrete, GPa; B is the compressive strength class of concrete, MPa.

It is important to note that equation (2) utilizes the average strength obtained from laboratory testing, whereas equations (3), (4), and (5) are based on specified strength values or strength grades - defined with a certain level of statistical reliability. In construction practice, concrete is considered to meet its specified strength or grade if the average compressive strength of molded specimens exceeds the specified value by a margin that accounts for concrete production variability.

This analysis highlights significant

discrepancies in elastic modulus values across different standard systems. Therefore, it is essential to establish empirical coefficients through practical experimentation, particularly using locally available aggregates. This study investigates the influence of two commonly used Vietnamese coarse aggregates - Ha Nam limestone and Hoa

Binh basalt - on the modulus of elasticity of normal-strength concrete. Additionally, the secant, chord, and initial moduli of elasticity are determined according to European, American, and Vietnamese standards to assess the accuracy of predictive formulas specified in various design standards.

2. Materials and methods

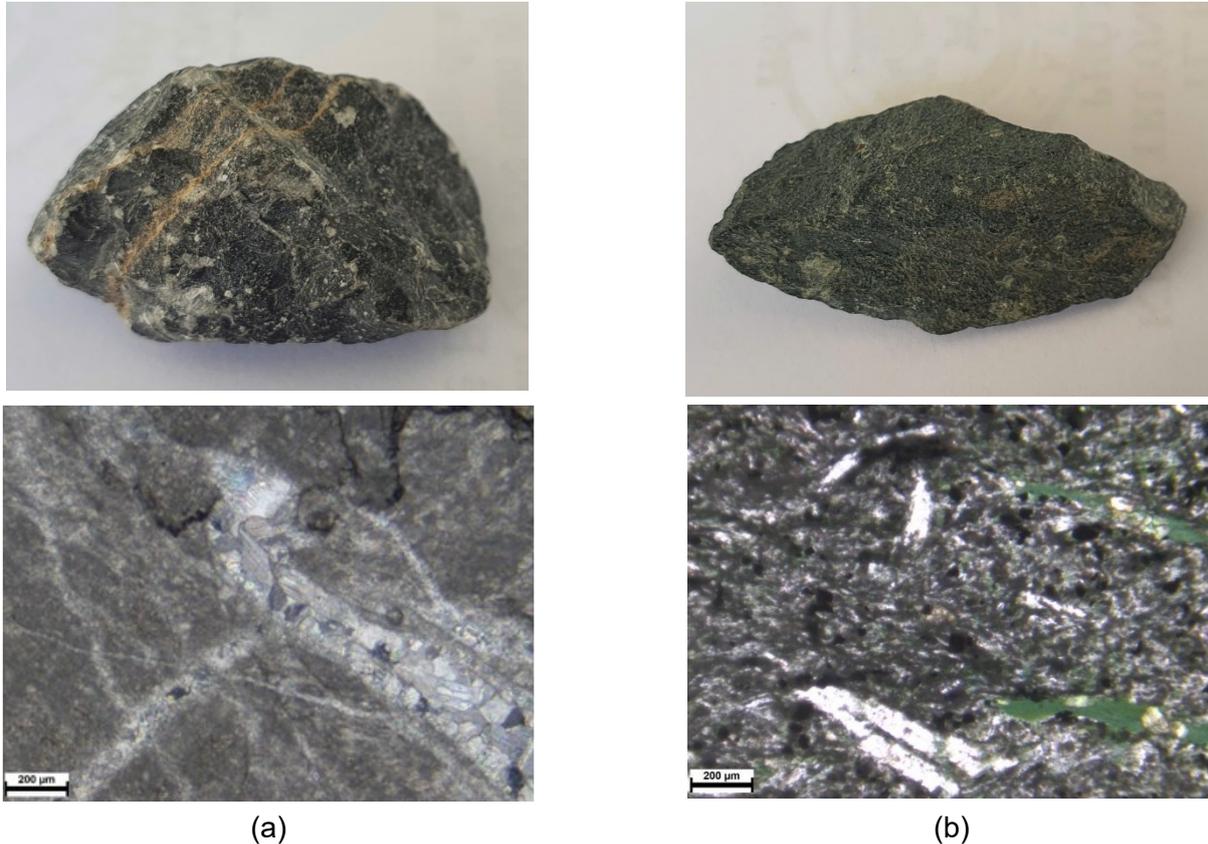


Fig 1. Normal view and petrographic thin section photos of (a) Ha Nam limestone and (b) Hoa Binh basalt under plane polarized light (1 Nicol)

2.1. Materials

This study employed two types of coarse aggregates: limestone sourced from Ha Nam Province and basalt obtained from Hoa Binh Province. Petrographic analysis of thin sections revealed distinct mineralogical and textural features for each rock type (Fig. 1).

The Ha Nam limestone exhibits a light grey coloration, a microcrystalline texture, and a blocky structural habit. It is marked by prominent fissures infilled with secondary calcite, indicative of post-depositional mineralization processes. The rock primarily comprises tightly compacted carbonate microcrystals that show signs of mechanical deformation. These microcrystals are transected

by calcite veins, suggesting a history of fracturing and fluid infiltration. Sutured grain boundaries are commonly observed, likely resulting from dissolution-driven metamorphism, contributing to the rock's intricate internal fabric. Accessory minerals include minor iron oxides, which may impart subtle coloration and reflect oxidative diagenetic conditions.

The basalt sample from Hoa Binh Province used in the experiments is characterized by a blue-gray hue and a blocky morphology, with a microcrystalline groundmass indicative of rapid cooling. The matrix consists predominantly of plate-shaped plagioclase microcrystals, interspersed with acicular (needle-like) crystals. In

certain zones, larger plate-shaped plagioclase phenocrysts are present, suggesting a complex cooling history involving multiple crystallization stages. Accessory colored minerals include microcrystalline amphibole and pyroxene, frequently associated with chlorite alteration. Under plane-polarized light (1 Nicol), chlorite (denoted as Cl in Fig. 1) exhibits a distinctive blue

coloration, facilitating its petrographic identification. Ore minerals are sparsely distributed throughout the sample, occurring as disseminated grains within the groundmass.

Table 1 presents the test results on the physical properties and gradation of the coarse aggregates, tested according to TCVN 7572:2006 [20].

Table 1. Properties of coarse aggregates

Items	Unit	Value for coarse aggregate	
		Ha Nam Limestone	Hoa Binh Basalt
Apparent particle density	g/cm ³	2.73	2.92
Bulk specific gravity (SSD)	g/cm ³	2.71	2.89
Bulk specific gravity (Dried)	g/cm ³	2.70	2.88
Water Absorption	%	0.4	0.4
Bulk density	kg/m ³	1400	1450
Voids	%	48.0	49.7
Aggregate crushing value	%	10	9
The smallest particle size Dmin	mm	5	5
The biggest particle size Dmax	mm	20	20
Cumulative retained on sieve			
40 mm		0	0
20 mm	%	8.8	9.8
10 mm		67.5	67.3
5 mm		98.5	99.2
Compressive strength	MPa	108.9	115.8

Table 2. Proportions of fresh concrete

Mixture ID	Material used for 1 m ³ fresh concrete, kg/m ³					
	Cement	Sand	Limestone agg.	Basalt agg.	Water	Chem. adm.
LC.1	330	776	1117	--	185	2.64
LC.2	395	745	1103	--	179	3.16
LC.3	480	706	1090	--	170	4.33
BC.1	330	820	--	1142	185	2.64
BC.2	395	791	--	1127	179	3.16
BC.3	480	751	--	1115	170	4.33

The fine aggregate used was the river sand from the Lo River with a specific gravity (dry) of 2.63 g/cm³, a bulk density of 1380 kg/m³, a water absorption of 0,5 wt.%, and a fineness modulus of 2.3.

We used PC40 Portland cement from the But Son cement plant with a specific gravity of 3.07 g/cm³, a specific surface of 3920 cm²/g, a standard consistency of 28.5 %, and initial and final setting

times of 120 min and 195 min. The compressive strengths of cement at 3 days and 28 days were 32.9 MPa and 52.1 MPa, respectively.

We also used SikaPlast-319H - a type F high-range water-reducing chemical admixture based on polycarboxylate, with a water reduction ability of 18,5%.

To investigate the influence of coarse aggregate type and test methods on the modulus

of elasticity, six concrete mixtures were prepared. Three of these mixtures incorporated Ha Nam limestone, and three used Hoa Binh basalt as the coarse aggregate (refer to Table 2). The water-to-cement (W/C) ratios ranged from 0.35 to 0.56, allowing for a comparative analysis under varying mix conditions.

2.2. Test methods

Concrete mixture was prepared using a free-fall laboratory mixer with a capacity of 50 dm³. Test specimens were cast in accordance with relevant standards (EN 12390-1:2021 [12], ASTM C192-25 [21], TCVN 3105:2022 [22]), including: cylindrical specimens (150 mm diameter x 300 mm height), cube specimens (150x150x150 mm), and prismatic specimens (150x150x600 mm). Each series consisted of three specimens. We cure the standard specimens under controlled laboratory conditions until the designated testing ages.

At 28 days, we determine the compressive strength for both cube and cylindrical specimens.

We also measure the modulus of elasticity of concrete using three standardized test procedures, including EN 12390-13:2021, ASTM C469-10, and TCVN 5726:2022.

We determine the stabilized secant modulus of elasticity $E_{c,s}$, in MPa, on cylindrical specimens 150x300 mm according to Method B of EN 12390-13:2021 [12] (Fig. 2a), as described below:

$$E_{c,s} = (\sigma_a^m - \sigma_p^m) / (\epsilon_{a,3} - \epsilon_{p,2}) \tag{6}$$

where: σ_a^m is measured stress corresponding to nominal upper stress $\sigma_a = f_c / 3$ (f_c is the compressive strength of concrete determined by testing companion specimens), MPa; σ_p^m is measured stress corresponding to nominal preload stress ($0.05MPa \leq \sigma_p \leq \sigma_b$, $0,10f_c \leq \sigma_b \leq 0,15f_c$), MPa; $\epsilon_{a,3}$ is the average strain at upper stress on the 3rd loading cycle, mm; $\epsilon_{p,2}$ is the average strain at preload stress on the 2nd loading cycle, mm.



(a)



(b)

Fig 2. Test setup for modulus elasticity test using cylindrical specimen (D150H300) according to EN 12390-13:2021 (a), and prism specimen (150x150x600) according to TCVN 5726:2022

According to ASTM C469-10 [23], we calculate the chord modulus of elasticity E_c , in MPa, as follows:

$$E_c = (S_2 - S_1) / (\epsilon_2 - 0,000050) \tag{7}$$

where: S_2 is the stress corresponding to 40 % of the ultimate load, MPa; S_1 is the stress corresponding to a longitudinal strain of 50 millionths, MPa; ϵ_2 is the longitudinal strain

produced by stress S_2 .

We used the prism specimens 150x150x600 mm to determine the initial modulus of elasticity E_b , in MPa, according to TCVN 5726:2022 [14] (Fig. 2b) as follows:

$$E_b = \sigma_1 / \varepsilon_{1y} \tag{8}$$

where: $\sigma_1 = P_1 / A$ is an increase in stress as the load increases from a supposed zero value (about 2 % of the ultimate load) to a load equal to 30 % of

the ultimate load, MPa; P_1 is the increase in load, N; ε_{1y} is the increase in strain corresponding to the load level $P_1 = 0,3P_e$, mm; P_e is the ultimate load, N;

It is evident that the three standard methods differ in specimen geometry and in the definition of the elastic deformation range, which may influence the measured modulus values.

3. Results and discussion

Table 3. Slump and unit weight of fresh concrete

Mixture ID	Slump, mm	Unit weight, kg/m ³
LC.1	120	2410
LC.2	120	2425
LC.3	130	2450
BC.1	120	2480
BC.2	130	2495
BC.3	130	2520

Table 4. Compressive strength of concrete

Mixture ID	Compressive strength on the cube specimen, R_{cub}		Compressive strength on the cylindrical specimen, R_{cyl}		R_{cub} / R_{cyl}
	Value, MPa	CoV ^a , %	Value, MPa	CoV ^a , %	
LC.1	35.6	4.5	31.7	2.2	1.12
LC.2	47.5	2.9	44.6	3.2	1.07
LC.3	56.9	2.7	55.1	2.0	1.03
BC.1	34.7	3.8	30.1	3.2	1.15
BC.2	47.9	3.0	44.3	2.1	1.08
BC.3	57.2	3.0	54.9	1.5	1.04

^aCoV is a within-series coefficient of variation

Test results on the properties of fresh concrete (Table 3) show that mixtures containing basalt aggregate have unit weights ranging from 2480 kg/m³ to 2520 kg/m³ - approximately 2.8 % higher than those incorporating limestone aggregate, which ranges from 2410 kg/m³ to 2450 kg/m³. This difference is primarily attributed to the higher density of basalt, which is about 6.6 % greater than that of limestone (see Table 1).

Compressive strength test results of concrete at 28 days, obtained from both cube and cylindrical specimens, are summarized in Table 4. Measured strengths ranged from 30.1 MPa to 57.2

MPa, with within-series coefficients of variation ranging from 1.5 % to 4.5 %. At a constant W/C ratio, the difference in compressive strength between concrete made with Ha Nam limestone and Hoa Binh basalt aggregates falls within the margin of statistical error. These results suggest that, in this study, the aggregate origin has a minimal influence on compressive strength, despite basalt showing slightly higher strength than limestone.

Table 4 also reveals that the ratio of cube to cylinder compressive strength varies with the overall concrete strength. At approximately 30

MPa, this ratio reaches 1.15, but it decreases with increasing strength, falling below 1.05 when the compressive strength exceeds 55 MPa.

Table 5, Table 6, and Table 7 present the modulus of elasticity of concrete at 28 days, determined according to EN 12390-13:2021,

ASTM C469-10, and TCVN 5726:2022, respectively. Each test series consisted of three specimens. The within-series coefficient of variation for the elastic modulus, ranging from 1.3 % to 4.6 %, indicate a high level of experimental consistency.

Table 5. Secant modulus of elasticity tested according to EN 12390-13:2021

Specimen ID	$\varepsilon_{a,3}$, 10^{-6} mm	σ_a^m , MPa	$\varepsilon_{p,2}$, 10^{-6} mm	σ_p^m , MPa	$E_{c,s}^i$, GPa	$E_{c,s}$, GPa	CoV, %
LC.1-1	87	3,2	287	9.5	31.5		
LC.1-2	78	3,2	270	9.5	32.8	31.5	4.3
LC.1-3	74	3,2	283	9.5	30.1		
LC.2-1	123	4,5	374	13.4	35.5		
LC.2-2	119	4,5	391	13.4	32.7	33.6	4.8
LC.2-3	124	4,5	396	13.4	32.7		
LC.3-1	128	5,5	432	16.5	36.2		
LC.3-2	122	5,5	412	16.5	37.9	37.5	3.0
LC.3-3	131	5,5	418	16.5	38.3		
BC.1-1	109	3,0	309	9.0	30.0		
BC.1-2	113	3,0	317	9.0	29.4	29.4	2.0
BC.1-3	96	3,0	304	9.0	28.8		
BC.2-1	130	4,4	413	13.3	31.4		
BC.2-2	122	4,4	387	13.3	33.6	32.5	3.4
BC.2-3	143	4,4	417	13.3	32.5		
BC.3-1	145	5,5	461	16.5	34.8		
BC.3-2	143	5,5	474	16.5	33.2	34.1	2.4
BC.3-3	163	5,5	485	16.5	34.2		

Note: See equation (6) for definition of σ_a^m , σ_p^m , $\varepsilon_{a,3}$, $\varepsilon_{p,2}$. $E_{c,s}^i$ and $E_{c,s}$ are the secant modulus of elasticity of i specimen and of a serie of specimens.

The experimental results demonstrate that, unlike compressive strength, the origin of the coarse aggregate significantly affect the modulus of elasticity. Depending on the concrete strength and the testing standard applied, the difference in elastic modulus between mixtures using limestone and basalt aggregates ranges from 3.4 % to 12.0 %. In all three testing methods, concrete incorporating Ha Nam limestone consistently exhibited a higher modulus of elasticity than that using Hoa Binh basalt. This effect may be attributed to the characteristics of the basalt used in the study, particularly its plate-shaped microstructure and the presence of chlorite alteration. Previous studies [24-25] have also demonstrated that weathering and alteration

processes reduce the stiffness of basalt. Consequently, the modulus of elasticity of both the rock and the concrete produced from it decreases relative to that of unaltered basalt.

Tables 5 and 6 show that the difference between the chord and the secant modulus of elasticity is less than 3.5 %. This small variation is likely result from the use of identical cylindrical specimens (150 mm in diameter and 300 mm in height) and the closeness in loading ranges, which vary from 10 % to 30 % of ultimate load for EN 12390-13:2021, and approximately from 4-7 % to 40 % for ASTM C469-10.

According to Table 7, the initial modulus of elasticity measured using TCVN 5726:2022 is consistently higher than both the secant and chord

moduli. This can be explained by the use of prismatic specimens (150×150×600 mm) and a lower initial loading range (approximately from 2 % to 30 % of the ultimate load). EN 1992-1-1:2004 provides equation (2) for estimating the secant modulus of elasticity, based on the compressive strength from cylindrical specimens. The

calculated value is subsequently adjusted to reflect the influence of the coarse aggregate type. As presented in Table 8, the estimated modulus for concrete containing limestone aggregate is lower than the experimental value, while the estimated modulus for concrete containing basalt aggregate exceeds the experimental result.

Table 6. Chord modulus of elasticity tested according to ASTM C 469-10

Specimen ID	S_1 , MPa	ε_2 , millionths	S_2 , MPa	E_c^i , GPa	E_c , GPa	CoV, %
LC.1-1	1.4	430	12.7	29.7		
LC.1-2	1.6	404	12.7	31.4	30.6	2.7
LC.1-3	1.5	413	12.7	30.9		
LC.2-1	2.2	532	17.8	32.4		
LC.2-2	2.1	493	17.8	35.4	33.8	4.6
LC.2-3	2.2	513	17.8	33.7		
LC.3-1	2.3	538	22.0	40.4		
LC.3-2	2.4	578	22.0	37.1	38.4	4.6
LC.3-3	2.3	574	22.0	37.6		
BC.1-1	2.1	391	12.0	29.0		
BC.1-2	2.0	409	12.0	27.9	28.7	2.4
BC.1-3	2.0	394	12.0	29.1		
BC.2-1	2.3	517	17.7	33.0		
BC.2-2	2.2	561	17.7	30.3	31.4	4.4
BC.2-3	2.2	552	17.7	30.9		
BC.3-1	2.7	623	22.0	33.7		
BC.3-2	2.6	621	22.0	34.0	34.3	2.6
BC.3-3	2.5	602	22.0	35.3		

Note: See equation (7) for definition of S_1 , S_2 , ε_2 . E_c^i and E_c are the chord modulus of elasticity of i specimen and of a serie of specimens.

The relative deviation in estimated modulus of elasticity for concrete using limestone aggregate reaches -7.7% with 10% reduction factor and up to 28.2 % with a 30 % reduction. For concrete using basalt aggregate, deviation can be as high as 36.6 %. To improve the accuracy of the estimation equation (2), empirical coefficients tailored to each aggregate type are proposed. Specifically, replacing the standard coefficient of 22,000 with 21,000 for limestone aggregate and 19,700 for basalt aggregate reduces the deviation between calculated and experimental values to within 3.4 %. These adjustments yield reductions of 4.5 % and 10.5 %, respectively - both smaller than the

deviation reported for South African granite aggregates [5].

The chord modulus of elasticity can be estimated according to ACI CODE-318-25 using equations (3) and (4), which require specified compressive strength (f'_c), rather than the average strength of the series of specimens. To meet the criteria for f'_c , the average strength of any three consecutive series must not be less than f'_c , and the average strength of any individual series must not be less than $f'_c - 3.5$ MPa when $f'_c \leq 35$ MPa, or less than $0.9f'_c$ when $f'_c > 35$ MPa. Table 9

presents the calculated chord modulus of elasticity value using equation (4), with f'_c taken as the strength of each specimen series taken as.

To assess the suitability of the proposed coefficients, the root mean squared error (RMSE) was calculated for each case. For the secant modulus of elasticity, estimation using equation (2) yielded RMSE values of 1.76 and 3.70 for concretes with Ha Nam limestone and Hoa Binh basalt aggregates, respectively. When applying the proposed factors of 21,000 and 19,700, the corresponding RMSE values decreased markedly to 0.74 and 0.71. Similarly, for the chord modulus of elasticity, equation (2) resulted in RMSE values of 3.43 and 1.71 for concretes with Ha Nam limestone and Hoa Binh basalt, whereas the use of the proposed factors of 5,250 and 4,810 reduced

the RMSE to 1.00 and 1.58, respectively. These findings demonstrate that the proposed preliminary coefficients substantially improve the accuracy of modulus of elasticity estimations.

The results in Table 8 show that those estimated using equation (4) are generally lower than experimental values for concrete incorporating both limestone and basalt coarse aggregates. Relative deviations between estimated and experimental values reach up to 13.5 % for limestone aggregate and 10.2 % for basalt aggregate. However, by replacing the standard coefficient of 4700 in equation (4) with empirically derived values equal to 5250 for limestone aggregate and 4810 for basalt aggregate, the relative deviations are reduced to less than 3.7 % and 6.1 %, respectively.

Table 7. Initial modulus of elasticity tested according to TCVN 5726:2022

Specimen ID	ϵ_{1y} , 10^{-6} mm	σ_1 , MPa	E_b^i , GPa	E_b , GPa	CoV, %
LC.1-1	342	10.7	31.2		
LC.1-2	332	10.7	32.2	32.1	2.5
LC.1-3	325	10.7	32.9		
LC.2-1	383	14.3	37.3		
LC.2-2	368	14.3	38.9	37.6	3.3
LC.2-3	392	14.3	36.5		
LC.3-1	421	17.1	40.6		
LC.3-2	433	17.1	39.5	40.5	2.2
LC.3-3	414	17.1	41.3		
BC.1-1	349	10.4	29.8		
BC.1-2	337	10.4	30.9	30.8	3.2
BC.1-3	327	10.4	31.8		
BC.2-1	402	14.4	35.8		
BC.2-2	394	14.4	36.5	35.6	3.0
BC.2-3	418	14.4	34.4		
BC.3-1	436	17.2	39.4		
BC.3-2	448	17.2	38.4	39.2	1.9
BC.3-3	431	17.2	39.9		

Note: See equation (8) for definition of ϵ_{1y} , σ_1 . E_b^i and E_b are the initial modulus of elasticity of i specimen and of a serie of specimens.

The values align with findings from study [9], which reported empirical coefficients for normal-strength concrete (from 20 MPa to 50 MPa) of 4871, 5118, 5204, 5532, and 6136 for concrete

using coarse aggregate commonly used in Santa Marta, Bogota, Cali, Cartagena/Barranquilla, and Medellin, respectively. This underscores the significant influence of aggregate characteristics

and the challenge of developing universal predictive formulas.

Overall, ACI CODE-318-25 estimation yields greater deviation than EN 1992-1-1:2004. This may be attributed to the difference in the exponent used in the strength-based calculation: ACI CODE-318-25 applies an exponent of $\frac{1}{2}$, whereas EN 1992-1-1 uses an exponent of $\frac{1}{3}$, which may better reflect the nonlinear relationship between compressive strength and elastic modulus for the aggregates studied.

According to TCVN 5574:2018, the initial modulus of elasticity is estimated using tabulated values corresponding to the designated concrete strength class. Compliance with strength class requirements involves the use of a required coefficient (k_{yc}), which accounts for variability in compressive strength between series. As specified in TCVN 10303:2014 [26], this coefficient depends on the between-series coefficient of variation (CV) of the concrete strength. For normal concrete, the typical range is from 1.07 to 1.43. In cases where statistical parameters are unavailable, a default value of 1.28 corresponding to a CV of 13.5 % is commonly adopted. Table 10 presents the

estimated initial modulus of elasticity values based on TCVN 5574:2018, using required coefficients of 1.07 and 1.28 for comparative analysis.

The data in Table 10 demonstrate that the estimated initial modulus of elasticity is sensitive to the required coefficient, which reflects the variation of concrete strength. When increasing from 1.07 to 1.28, the difference between the estimated and experimental values reaches 1.9 GPa. At a required coefficient of 1.28, relative deviation between estimated and experimental values increases with compressive strength from -3.4 % to -11.0 % for concrete with limestone aggregate, and from -0.2 % to -7.9 % for concrete with basalt aggregate. These results suggest that for high-strength concrete, experimentally determined values of initial modulus of elasticity are preferable accurate.

It should be noted that this study was conducted using only two aggregate types and a limited number of samples. Therefore, the proposed empirical coefficients are intended for reference only. Future large-scale studies are needed to refine these values and enhance predictive reliability.

Table 8. Experimental and estimated secant modulus of elasticity

Mixture ID	$E_{c,s}$, GPa	R_{cyl} , MPa	Estimated modulus of elasticity according to eq. (2) ^a			Estimated modulus of elasticity with proposed factor ^b	
			E_{cm} , GPa	E_{cm}^{corr} , GPa	RD ^c , %	E_{cm}^a , GPa	RD ^c , %
LC.1	31.5	31.7	32.3	29.1	-7.7	30.8	-2.2
LC.2	33.6	44.6	36.2	32.6	-3.0	34.6	3.0
LC.3	37.5	55.1	38.9	35.0	-6.7	37.1	-1.1
BC.1	29.4	30.1	31.8	38.1	29.7	28.4	-3.4
BC.2	32.5	44.3	36.3	43.6	34.2	32.5	0.0
BC.3	34.1	54.9	38.8	46.6	36.6	34.8	2.1

^a E_{cm} is calculated according to eq. (2), E_{cm}^{corr} is the reduced E_{cm} by 10 % for concrete with limestone aggregate, and increased E_{cm} by 20 % for concrete with basalt aggregate.

^b E_{cm}^a is estimated modulus of elasticity based on eq. (2) with factors 21000 and 19700 for concretes with limestone and basalt aggregates, respectively.

^c Relative deviation is calculated as $RD = (E_{cm} - E_{c,s}) / E_{c,s}$, %

Table 9. Experimental and estimated chord modulus of elasticity

Mixture ID	E_c , GPa	R_{cyl} , MPa	Estimated modulus of elasticity according to eq. (4) ^a		Estimated modulus of elasticity with proposed factor ^b	
			E_c^{4700} , GPa	RD, %	E_c^a , GPa	RD, %
LC.1	30.6	31.7	26.5	-13.5	29.6	-3.4
LC.2	33.8	44.6	31.4	-7.1	35.1	3.7
LC.3	38.4	55.1	34.9	-9.1	39.0	1.5
BC.1	28.7	30.1	25.8	-10.2	26.4	-8.1
BC.2	31.4	44.3	31.3	-0.4	32.0	2.0
BC.3	34.3	54.9	34.8	1.5	35.6	3.9

^a E_c^{4700} is calculated according to eq. (4).

^b E_c^a is estimate modulus of elasticity based on eq. (4) with factor 5250 and 4810 for concretes with limestone and basalt aggregates, correspondingly.

Table 10. Experimental and estimated initial modulus of elasticity

Mixture ID	E_b , GPa	R_{cub} , MPa	The estimated modulus of elasticity when $k_{yc} = 1.07$		The estimated modulus of elasticity when $k_{yc} = 1.28$	
			$E_b^{1.07}$, GPa	RD, %	$E_b^{1.28}$, GPa	RD, %
LC.1	30.6	35.6	26.5	-13.5	29.6	-3.4
LC.2	33.8	47.5	31.4	-7.1	35.1	3.7
LC.3	38.4	56.9	34.9	-9.1	39.0	1.5
BC.1	28.7	34.7	25.8	-10.2	26.9	-6.1
BC.2	31.4	47.9	31.3	-0.4	32.7	4.1
BC.3	34.3	57.2	34.8	1.5	36.4	6.1

4. Conclusions

The experimental results on the modulus of elasticity of concrete with compressive strengths ranging from approximately 30 MPa to 55 MPa, using Ha Nam limestone and Hoa Binh basalt coarse aggregates, and evaluated according to European, American, and Vietnamese standards, lead to the following conclusions:

- The modulus of elasticity of concrete depends on the geological origin of coarse aggregate. At comparable compressive strengths, concrete incorporating Ha Nam limestone consistently exhibits a higher modulus of elasticity than that made with Hoa Binh basalt.

- The relative difference between the secant modulus of elasticity (EN 12390-13:2021) and the chord modulus of elasticity (ASTM C469-10) was

less than 3.5 % for the tested concrete, indicating close agreement between these two test methods.

- Modulus of elasticity values estimated using European, American, and Vietnamese standards showed differing levels of deviation from experimental results. For concrete with limestone aggregate, greater deviation was observed with American and Vietnamese standards. In contrast, concrete incorporating basalt aggregate showed higher deviation when estimated using European standards.

- To enhance predictive reliability, the study recommends using preliminary coefficients of 21,000 and 19,700 for Ha Nam limestone and Hoa Binh basalt, respectively, when applying EN 1992-1-1. For ACI CODE-318-25 estimations, recommended preliminary factors are 5,250 and

4,810, respectively.

- For applications requiring high accuracy, the modulus of elasticity should be determined through direct experimental testing rather than relying solely on the standard estimation equations.

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