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Optimizing Solar Energy Systems: Enhancing Photovoltaic Efficiency through Mirror Integration and Parameter Optimization

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Abstract: This study presents a geometric optimization approach to enhance photovoltaic PV efficiency by integrating a ground-mounted mirror. The method aims to identify optimal values for the panel tilt angle β, mirror tilt angle α, and mirror length L that maximize reflected irradiance onto the PV surface without obstructing direct sunlight. A mathematical expression is developed from geometric constraints to define the optimal mirror configuration. The convergence history of the numerical solution is analyzed, and a sensitivity study on design variables is provided. An energy estimation based on clear-sky conditions in Khouribga shows that the optimized mirror configuration can increase annual irradiance from 1680 kWh/m² (no mirror) to 1895 kWh/m². The optimal setup is achieved with β = 30°, α = 15.1°, and L = 2.45 m. The proposed method offers a simple, accessible design tool suitable for practical use without the need for complex simulation software.

Keywords: Photovoltaic systems; Mirror integration; Tilt angle optimization; Solar irradiance; Energy efficiency.

Nomenclature

α: Mirror tilt angle (°).
β: Photovoltaic (PV) panel tilt angle (°).
L: Length of the mirror (m).
Lmus: Maximal useful mirror length (m)
h: Solar elevation angle (°).
hmin: Minimum solar elevation angle at solar noon (°).
l: Length of the PV panel (m).
Xref: Reflected sunlight coverage on the PV panel (m).
NOCT: Nominal Operating Cell Temperature (°C).
Irradiance: Solar energy incident per unit area (W/m²).

1. Introduction

The global demand for energy is experiencing unprecedented growth, driven by population expansion, industrialization, and urbanization. According to the International Energy Agency (IEA), global energy consumption increased by approximately 4.6% in 2021, marking one of the highest annual increases since 2010 [1].

This surge in energy demand places immense pressure on existing energy infrastructure and resources, prompting urgent calls for sustainable solutions. As traditional fossil fuel reserves dwindle and environmental concerns escalate, transitioning to more sustainable energy sources become imperative to ensure energy security and reduce greenhouse gas emissions [2].

In light of these challenges, renewable energy sources are becoming increasingly vital both now and in the future. They offer a path toward decarbonizing the energy sector while simultaneously meeting the growing energy demands of the global population. The United Nations (UN) has recognized renewable energy as a cornerstone in achieving its Sustainable Development Goals, particularly Goal 7, which aims to ensure access to affordable, reliable, sustainable, and modern energy for all [3]. The ongoing advancements in renewable technologies, coupled with supportive policies, position renewables as a key player in addressing climate change and fostering economic growth, paving the way for a sustainable energy future [4].

Among the various renewable energy sources, solar energy stands out as one of the most promising due to its vast potential and decreasing costs. The International Renewable Energy Agency (IRENA) highlights that solar power capacity has grown significantly, with solar photovoltaics (PV) emerging as a leading technology in the renewable energy landscape [5].

The ability of solar energy systems to be deployed at various scales—from residential rooftops to large utility-scale solar farms—offers flexibility and accessibility, making it a cornerstone of global energy strategies [6]. Additionally, the decreasing cost of solar technologies has made them competitive with traditional energy sources, further encouraging widespread adoption [7].

As solar energy usage expands, optimizing the performance of photovoltaic (PV) systems becomes critical to maximizing their efficiency and output. Performance optimization not only enhances the economic viability of solar installations but also addresses the intermittency and variability associated with solar energy production [8]. To this end, research into innovative methods such as optical enhancements and reflector integration has gained traction. Mirrors are often used to enhance solar capture in PV systems without tracking mechanisms [9]. This approach underscores the importance of performance optimization in developing effective and sustainable solar energy solutions, ultimately contributing to a more resilient energy future.

Recent advancements in photovoltaic (PV) system efficiency have increasingly emphasized the integration of reflectors, particularly mirrors, as a means to enhance solar energy capture. This innovative approach aims to optimize key parameters, including the tilt angles of both the PV panels and the mirrors, as well as the mirrors' length, to maximize irradiance and output power. By addressing challenges such as land space limitations and the reduction of efficiency due to elevated panel temperatures, researchers are paving the way for more effective solar energy solutions.

One prominent study by Nacer et al. (2021) focuses on the geometric optimization of flat reflectors to improve photovoltaic performance. It investigates various tilt angles and reflector geometries to enhance the irradiance on the PV surface, allowing for increased energy capture without the need for complex tracking systems. The findings emphasize the significance of precise geometrical adjustments to maximize efficiency under specific climatic conditions [9].

Another research effort by Med. Karimzadeh Kolamroudi et al (2022) explores the application of reflecting mirrors in low-concentration PV systems, demonstrating substantial improvements in output power. By testing different configurations of mirror tilt and positioning, this study identifies the optimal setup for enhancing incident solar radiation, highlighting the potential of simple mirror reflectors for energy maximization [10].

Additionally, Agrawal M and Chhajed P (2022) analyze the performance of PV modules integrated with reflectors, focusing on optimizing the tilt angles of both the module and the reflector. The investigation reveals that the optimized orientation significantly boosts energy output, making it a viable solution for regions characterized by variable solar irradiance [11].

Furthermore, Malik P and Chandel S (2020) examine the performance enhancement of multi-crystalline silicon PV modules using mirror reflectors under the unique climatic conditions of the Western Himalayas. This study optimizes reflector tilt and distance to increase solar energy absorption, accounting for high-altitude solar intensity and extreme temperatures [12].

Lastly, Tabasi et al. (2019) conduct a detailed investigation into the performance optimization of a PV panel integrated with a reflecting mirror, revealing critical factors such as the mirror's tilt angle, length, and the panel's orientation. The study systematically adjusts these parameters to enhance the irradiance received by the PV panel, ultimately improving overall efficiency. It also addresses the balance between increased irradiance and potential overheating, recommending cooling strategies to mitigate temperature-related efficiency losses [13].

The principal contribution of this work lies in establishing a geometrically driven optimization framework that couples real solar elevation data with panel and mirror configurations to identify optimal parameters enhancing PV system efficiency. By analytically defining and solving a trivariate objective function involving mirror tilt, panel tilt, and mirror length, this study presents a flexible method adaptable to varying climatic conditions and locations. The inclusion of practical boundaries, seasonal configurations, and the introduction of a critical reflection point offers a novel approach to solar harvesting enhancement, particularly for flat-surface installations in high-insolation regions, Khouribga, Morocco in our study.

2. Optimization methods

The integration of mirrors into photovoltaic (PV) systems has been the subject of extensive research aimed at enhancing energy capture and overall power output. In the early study by Ahmad and Hussein (2001), the authors employed a performance analysis approach to optimize the orientation of photovoltaic modules with reflectors by testing various tilt scenarios for both the PV panels and the mirrors, effectively identifying the most beneficial angles for maximizing energy output. Following this, Abdallah and Badran (2008) examined the performance of multi-crystalline silicon photovoltaic modules using mirror reflectors, focusing on the optimization of mirror tilt and placement distances to improve solar absorption under the distinct climatic conditions of the Western Himalayas. The research by Tabasi et al. (2019) expanded on these concepts, conducting a detailed investigation into the performance optimization of PV panels integrated with reflecting mirrors, where they systematically adjusted parameters such as mirror tilt, length, and panel orientation to enhance the irradiance captured by the PV system. Additionally, Kolamroudi and Ilkan (2022) explored low-concentration PV systems, employing reflecting mirrors and assessing different configurations of mirror tilt and positioning to determine optimal setups that maximize output power. In the study titled "Increase power output and radiation in photovoltaic systems by installing mirrors," researchers investigated the techniques used to enhance incident solar radiation on PV panels, emphasizing the identification of optimal mirror tilt angles and placements to achieve increased energy output. Most recently, Nacer et al. (2021) conducted a geometrical optimization study for PV installations equipped with flat reflectors, utilizing plane of array estimations to determine the most effective combinations of panel tilt and mirror parameters for enhancing solar energy capture. Collectively, these studies highlight the critical importance of optimizing system parameters—such as panel tilt, mirror tilt, and mirror length—to maximize the efficiency of photovoltaic systems utilizing mirrors for improved energy production [9],[10],[11],[12],[13],[14].

To initiate the determination of L, we employed boundary conditions under which the PV panel's tilt angle is set at 90 degrees, signifying a perpendicular orientation to the incoming sunlight, and the mirror's tilt angle corresponds to the lowest value of the daily maximal elevation angle throughout the year. This configuration served as the foundation for estimating L boundaries, aligning the mirror to optimize energy capture.

The determination of beta, representing the PV panel's tilt angle when not integrated with the mirror, was approached independently, as the choice in our study was taking into account the direct sunlight benefit too and not only the reflection, therefore we took the seasonal and annual optimal panel tilt from the study of Mohamed Nfaoui et al for the studied region Khouribga city Morocco [15].

The parameter alpha, signifying the mirror's tilt angle, was derived geometrically. We utilized triangular equations, skillfully linking the three parameters - L, beta, and alpha - into a single equation. This approach facilitated the calculation of alpha for a given set of values of L and beta, thus achieving a harmonized geometrical solution conducive to optimal energy capture.

2.1. Different possible cases

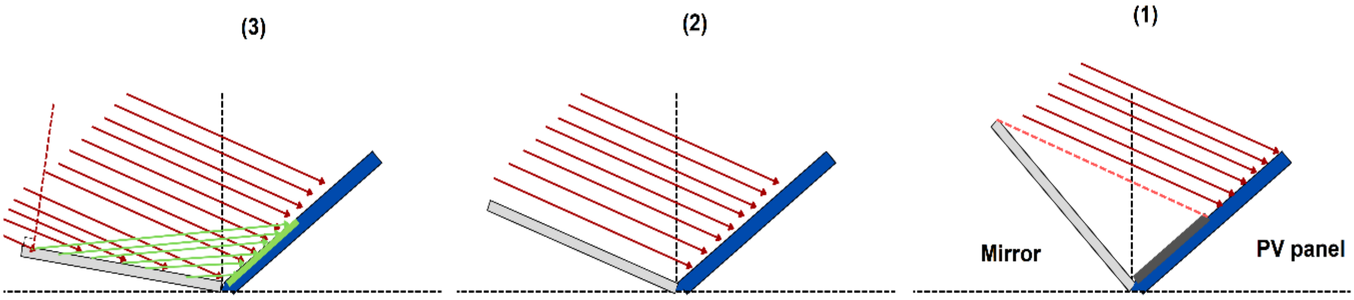

Fig. 1. The different possible cases of mirror effect (Shadow/reflection) on the panel for the same Elevation angle

The PV panel and the integrated mirror can exhibit different tilt angles. Consequently, in a pragmatic classification framework, three distinct cases of reflection and shadow interaction shown in Fig. 1.

Case 1: In this scenario, as shown in Fig. 2, the mirror shades the PV panel. This situation is not aligned with our intended outcome, as it adversely impacts system efficiency. Therefore, meticulous investigation is imperative to establish the tilt angle intervals within which shadowing effects manifest, with the ultimate objective of avoiding these intervals.

Case 2: Here, the mirror's positioning results in neither reflection nor shadow cast upon the PV panel. This condition yields no discernible impact on the incident irradiance received by the panel throughout the day. It essentially signifies the transition from cases involving shadowing to cases involving reflection. This transition When the mirror makes shadow on the panel.

As it is shown in Fig. 3, the mirror creates shadow on the panel, where xsh is the shadow length on the panel, h is the solar elevation angle, β, α, and L, are the panel tilt angle, the mirror tilt angle, and the length of the mirror respectively.

Case 3: This represents the focus of our research, where the mirror's positioning engenders sunlight reflection onto the PV panel. Here, the investigation assumes greater complexity as we aim to ascertain the optimal values for the controlled parameters α, β, and L in order to maximize sunlight reflection on the panel.

Within this realm, we must account for a fourth parameter, the solar elevation angle, denoted as "h," which remains beyond our control. Hence, the initial focus centers on delineating the intervals for the uncontrolled variable, the solar elevation angle, as it simplifies subsequent calculations. We establish the boundaries for the solar elevation angle as:

$hmin < h < hmax$

These boundary conditions serve as the foundational framework within which the remaining parameters are optimized to achieve maximal sunlight reflection on the PV panel.

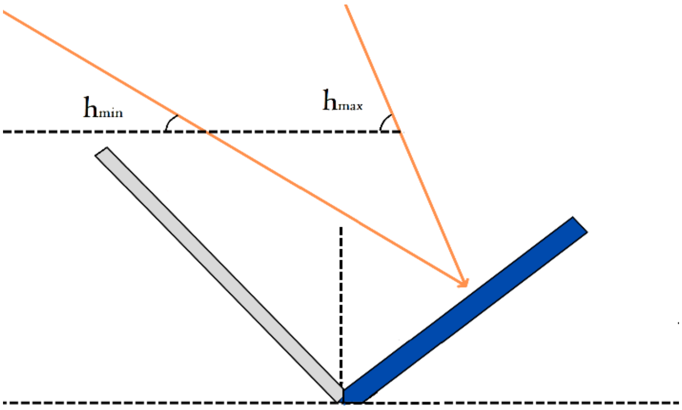

Fig. 2. The interval of the solar elevation angle

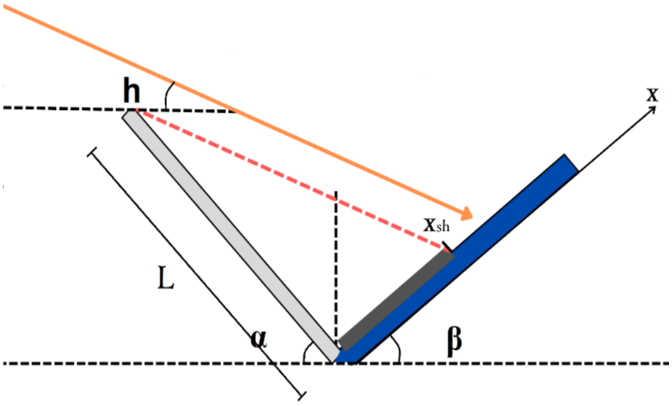

Fig. 3. The optical schematics of case 1

For the aim of the determination of this case's interval boundaries, we will consider the two boundaries as:

2.1.1. Complete shadow on the panel

This specific case is determined by selecting the minimal solar elevation angle (h) while keeping α, β, and L at predefined values. It's important to highlight that as the sun's elevation increases, the photovoltaic panel receives more irradiance. Conversely, as the sun's elevation decreases, Fig. 4. illustrates that more shadow is cast on the panel.

That leads to the following equations:

$\mathrm{Tan}(\theta) = l/L$

$(\pi/2-\alpha) + \theta + h_{min} = \pi/2$

which leads to:

$\theta = \alpha - h_{min}$

$\boldsymbol{\alpha = h_{min} + \tan^{-1}(l/L)}$

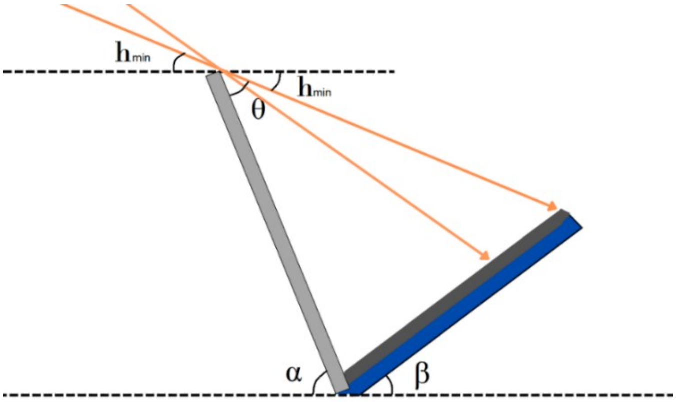

Fig. 4. The optical schematics of the case with a complete shadow on the panel

2.1.2. No shadow on the panel, but also no sunlight reflection

Which is represented by the case where θ=0, which means $\alpha = h_{min}$.

As illustrated in Fig. 5, the mirror is observed to exhibit neither the phenomenon of reflecting incident sunlight onto the photovoltaic panel nor that of creating shadows upon it.

This case can be only at times when α=h, which can let us separate between the two situations shadow/mirror reflection, in this way:

α < h: the mirror reflects sunlight onto the panel

α > h: the mirror shades on the panel

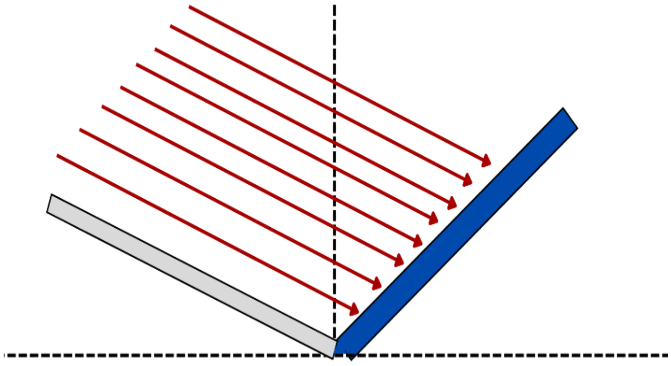

Fig. 5. The optimal schematic of case 2

2.1.3. When the mirror makes sunlight reflection on the panel

Fig. 6 visually demonstrates that in Case 3, the photovoltaic panel effectively captures reflected sunlight originating from the integrated mirror. This scenario, which aligns with the central theme of our study, will remain the sole focus of our investigation throughout the ensuing sections of this research paper.

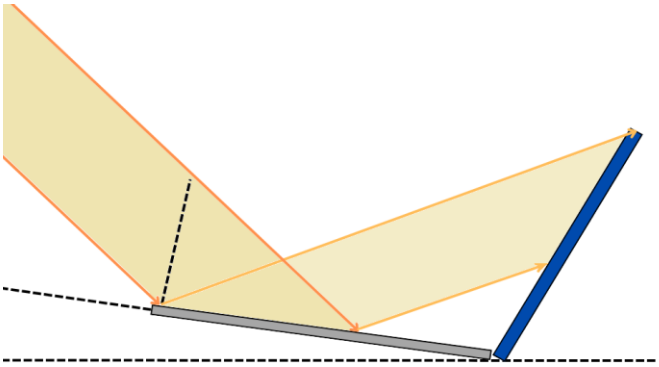

Fig. 6. The optimal schematic of case 3

2.2. Determination of the optimal values for α, β and L

2.2.1. A boundary condition for $L=L_{mus}$ (Maximal useful mirror surface)

In this study, the concept of the Maximal useful surface (denoted as Lmus) refers to the longest mirror length that can contribute to optimizing the photovoltaic (PV) system's performance. This length represents the limit beyond which further increases in the mirror size yield no additional benefit in terms of energy capture and system efficiency. As the mirror length extends beyond Lmus, the reflected irradiance fails to reach the panel.

The maximal useful mirror length is thus a critical component in designing the PV system. Beyond Lmus, any additional mirror length results in wasted material and unnecessary cost without any increase in energy capture. Therefore, determining the optimal value of Lmus is essential to ensuring that the system design remains efficient, cost-effective, and optimized for maximal energy output.

Lmus is particularly applicable for flat surfaces such as flat roofs or large fields where space is available, and mirrors can be deployed without concern for terrain limitations.

This maximal useful mirror length can be calculated using the boundary condition where the PV panel tilt angle is perpendicular to the ground (β = 90°) and the mirror's tilt angle is α = 0°. Additionally, the solar elevation angle is set to its noon's minimum value (hmin), typically occurring around midday during the year.

In Fig. 7 we provide a geometric representation of the trajectory of sunlight in this boundary condition. This illustration helps us understand how sunlight is reflected by the mirror and directed onto the photovoltaic panel.

From Fig. 7 we derive the relationship:

$\tan(h_{min}) = l/L$

where "l" represents the length of the photovoltaic panel (l), and "L" denotes the length of the mirror (L). With this relationship in hand, we can now determine the optimal mirror length, "Lmus." This value is calculated using the equation:

$$L_{mus}=\frac{l}{\tan(h_{min})} \quad (1)$$

Where: l is the length of the PV panel,

hmin is the minimum solar elevation angle, typically occurring around noon when sunlight is strongest.

This value of Lmus represents the largest useful surface area of the mirror that can effectively enhance the PV system's performance.

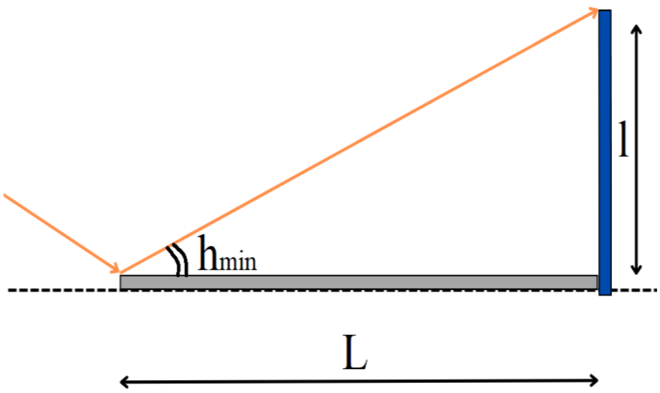

Fig. 7. The optimal schematic of reflected sunrays in the boundary condition β = 90°, α = 0°, hmin

2.2.2. Optimization problem

The optimization problem addressed in this study focuses on improving the optical performance of a PV system equipped with a static ground mirror. The target is to determine the optimal geometric configuration that maximizes the effective solar energy reflected onto the PV panel under the following critical solar elevation conditions:

A. Decision variables

The three parameters subject to adjustment are:

- α: mirror tilt angle (∘)
- β: PV panel tilt angle (∘)
- L: mirror length (m)

B. Objective function

The objective is to maximize the reflected sunlight that reaches the PV panel. This is achieved by ensuring that the reflected coverage on the panel surface, denoted Xref, equals the panel length l, under the limiting solar elevation angle hmin, typically occurring at solar noon during winter.

Thus, the objective function is:

Maximize X_{ref} ; X_{ref} (α,β,L)

Subject to the condition:

$X_{ref}=l$ when $h=h_{min}$

From the geometric analysis of the reflection path (refer to Fig. 8), the boundary condition leads to the following governing equation (2)

C. Constraints

The optimization process is subject to several physical and geometric constraints. First, the mirror tilt angle α must remain within the interval (0 < α < hmin) to avoid shading the panel and to maintain reflection effectiveness.

The panel tilt angle β is constrained within 0° < β < 70°, beyond which minimal gains in reflection are observed, and the direct sunlight onto the (PV) panel start to decrease.

The mirror length L is bounded by (0 < L ≤ Lmus). These constraints ensure realistic, efficient, and implementable configurations for real-world flat-roof systems.

To summarize, the mathematical formulation of the optimization problem:

Find (α,β,L):

Such that: Xref = l at h = hmin

Subject to:

$$\begin{cases} 0^\circ<\alpha<h_{min} \\ 0^\circ<\beta<70^\circ \\ 0<L<L_{mus} \end{cases}$$

This formulation defines a constrained nonlinear system with interdependent geometric variables, and it is solved numerically in the next section.

2.2.3. The optimization algorithm provided in Matlab software

The optimization process was implemented in MATLAB using a custom brute-force grid search algorithm based on the meshgrid function. This approach systematically evaluates all possible combinations of the decision variables (α, β, L) over predefined intervals within the feasible design space. For each triplet (α, β, L), the geometric reflection condition was tested to identify the configurations that satisfy the optimization constraint. This algorithm does not rely on any built-in solver such as

This method offers the advantage of capturing the complete solution space without being sensitive to initial conditions or local extrema. Moreover, it allows clear visualization of how the solution behaves across the parameter ranges, and supports the sensitivity analysis detailed later in Section 4.1.2.

Fig. 8 illustrates the geometric configuration of the photovoltaic (PV) system, highlighting the relationships between the PV panel, mirror, and incident sunlight. The figure emphasizes the angles and distances involved in reflecting sunlight from the mirror onto the panel under specific solar conditions, which are essential for optimizing the system’s performance.

In Fig. 8 the trajectory of sunlight is illustrated as it strikes the mirror and is reflected onto the photovoltaic (PV) panel. The incoming sunlight reaches the mirror at a solar elevation angle h, which varies throughout the day and across different seasons. This angle h is critical in determining the optimal tilt angle α of the mirror, which directs the reflected sunlight toward the PV panel.

After reflection, the sunlight covers a length on the panel, referred to as X_{ref}, representing the extent of the reflected surface on the panel. The PV panel is positioned at a tilt angle β to ensure it captures the full length of X_{ref}. The boundary condition $X_{ref} = l$ (where l is the panel length) ensures that the entire PV panel surface is illuminated by the reflected sunlight, maximizing energy capture.

For optimal configuration, the system considers h_{min}, the minimum solar elevation angle observed at noon over the course of the year.

Maintaining h in its lowest value at noon, we ensure minimal sunlight reflection is wasted through the year, and consequently no material waste.

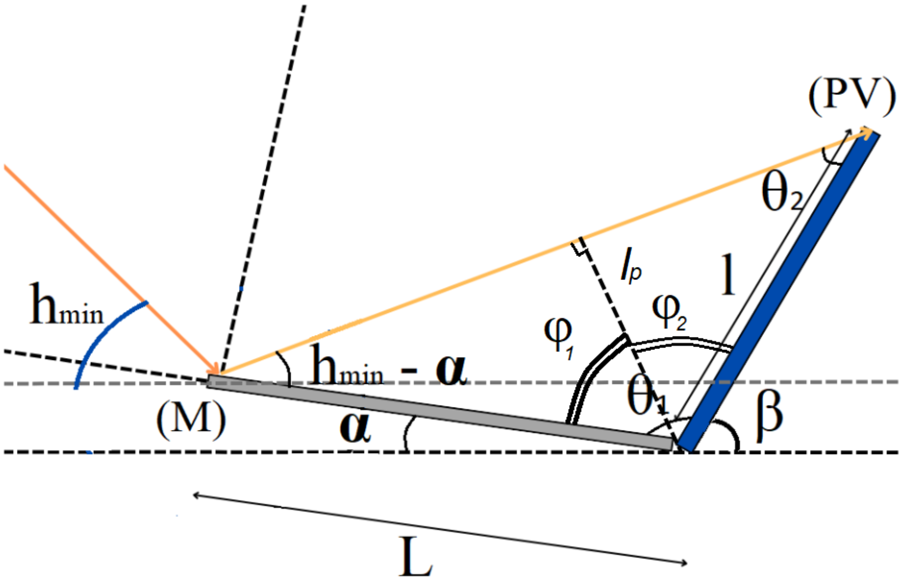

Fig. 8. The optimal schematic of reflected sunrays on the PV panel in case 3 in full panel coverage by the sunlight reflection at $h=h_{min}$

Adjusting the system to the lowest value of h at noon ensures the minimal shading effect into the panel through the year.

Also, this consideration ensures that the full reflection happens during the winter in low ambient temperature, while less sunlight is reflected to the panel in the summer when the panel is already overheated.

We obtain from Fig. 8:

$$\begin{cases} \alpha+\beta+\theta_2=\pi \\ h_{min}-\alpha+\theta_2+\theta_1=\pi \end{cases}$$

And:

$$\begin{cases} \theta_2=\varphi_1+\varphi_2 \\ h_{min}-\alpha+\varphi_1=\dfrac{\pi}{2} \\ \theta_1+\varphi_2=\dfrac{\pi}{2} \end{cases}$$

Where:

$$\begin{cases} \varphi_1=\dfrac{\pi}{2}+\alpha-h_{min} \\ \varphi_2=\dfrac{\pi}{2}-\theta_1 \end{cases}$$

Consequently:

$$2\alpha+\beta-h_{min}-\theta_1=0$$

Also from Fig. 8:

$$\begin{cases} \sin(\theta_1)=\dfrac{l_p}{l} \\ \sin(h_{min}-\alpha)=\dfrac{l_p}{L} \end{cases}$$

Therefore:

$$\theta_1=\sin^{-1}\left(\frac{l_p}{l}\right)$$

And: $$l_p=L\cdot \sin(h_{min}-\alpha)$$

Then we obtain equation (2):

$$L_{op}=l\cdot\frac{\sin\left(2\alpha_{op}+\beta_{op}-h_{min}\right)}{\sin\left(h_{min}-\alpha_{op}\right)} \quad (2)$$

Where: α_{op} is the optimal mirror tilt angle.

L_{op} is the optimal mirror length.

β_{op} is the optimal panel tit angle.

h_{min} is the lowest value of the solar elevation at noon (around 13h00 GMT) through the year.

To solve equation (2), referred to as the optimization function, we employ MATLAB's numerical computation capabilities. Since the system has multiple possible solutions based on different combinations of L, α, and β, MATLAB is well-suited to handle the complexity of this multi-variable equation. By utilizing MATLAB's numerical solvers, we explore the solution space comprehensively, identifying feasible configurations that maximize sunlight reflection onto the photovoltaic (PV) panel. The optimization function is programmed in MATLAB to account for varied values of L, α, and β, producing a range of possible solutions that achieve optimal system performance.

The MATLAB simulation generates a 3D graph that visualizes the solution space, illustrating how different values of L, α, and β interact to produce optimal configurations. This 3D representation provides insight into the combinations of these parameters that maximize sunlight capture in the PV system. Additionally, MATLAB outputs a table that summarizes various possible solutions, detailing specific values for L, α, and β that meet the optimization requirements. By reviewing this table, we can identify the most efficient configuration, taking into account the environmental and structural constraints of the system.

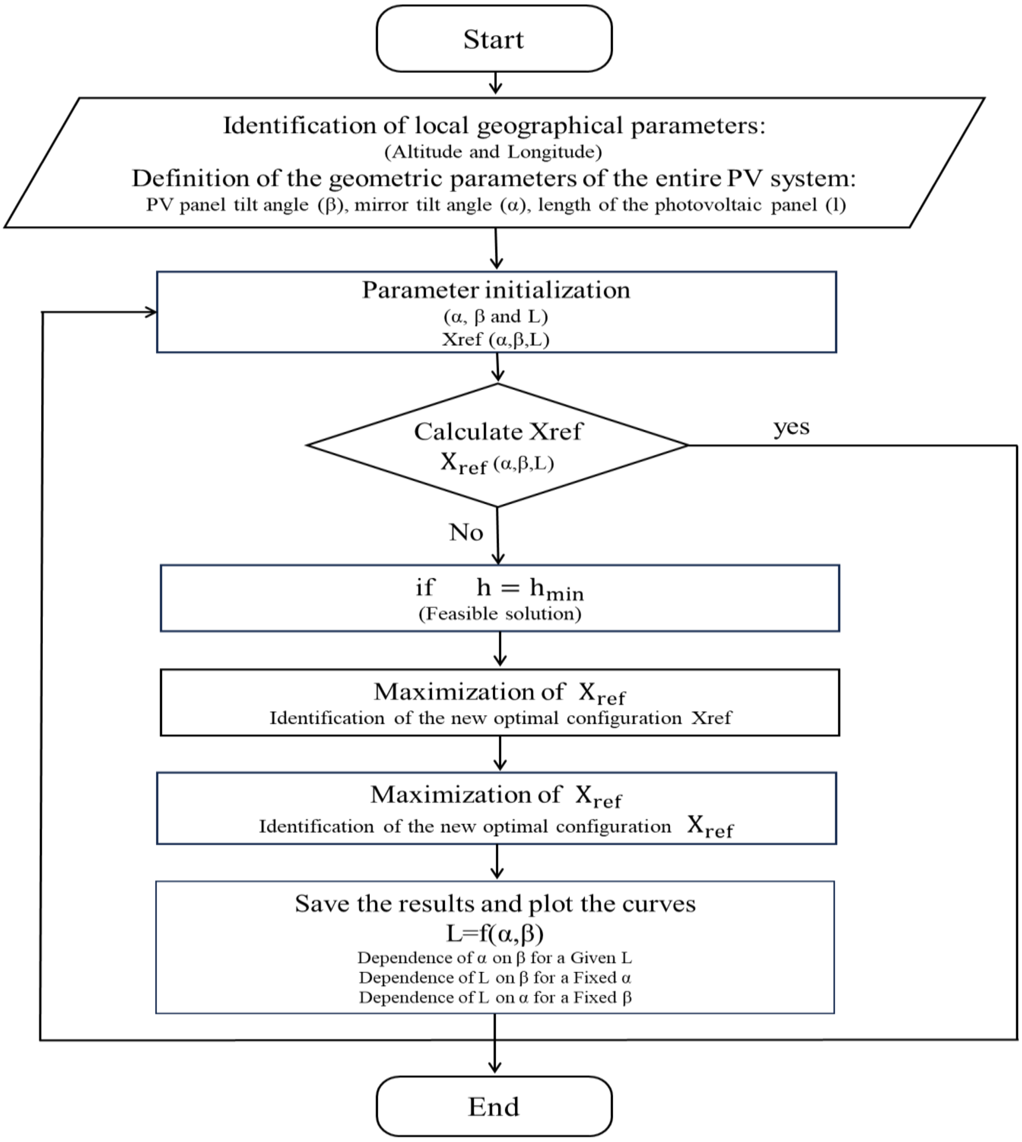

Fig. 9. Flowchart of the numerical optimization process for identifying valid (α, β, L) configurations

Fig. 9 summarizes the numerical procedure used in this study, from defining parameter ranges to identifying valid combinations that satisfy the reflection condition for optimal photovoltaic performance.

3. Studied location

This study specifically focuses on the region of Khouribga [16], Morocco, where unique solar conditions influence the design of the PV system. We use the defined optimal tilt angle for PV panels without mirrors as a baseline, adjusting the mirror tilt α and length L accordingly to align with Khouribga city's solar elevation patterns. By customizing the MATLAB simulation with regional parameters, the solutions derived are both practical and optimized for local conditions, enhancing the PV system's performance in real-world applications.

4. Results and discussion

4.1. System optimal configuration

To determine the optimal design for our study, it is imperative to establish the specifications of the photovoltaic (PV) panel and the characteristics of its location. In this research, we have based our calculations on the specifications of a commercially available solar cell, namely LG NeON 2 LG355N1C-N5 [17]. Fig. 10 provides a schematic representation of the panel's dimensions, as well as the connectivity between its constituent cells (refer to Fig. 10 for details).

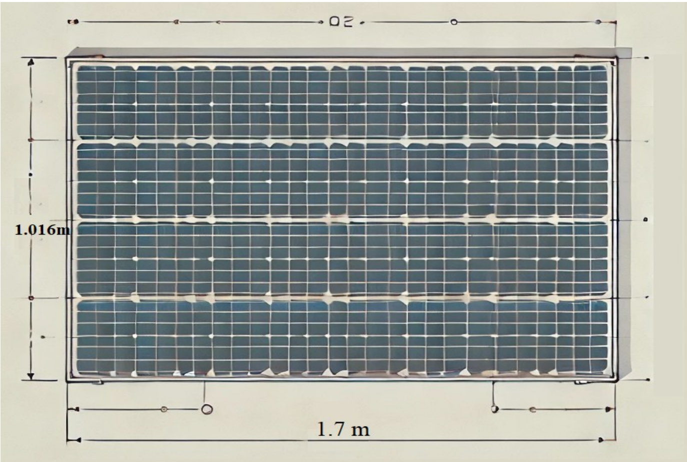

Fig. 10. Dimensions of the panel and the connection type between the cells

For a more comprehensive understanding, we have collated the specific details of the PV module in Table 1.

Regarding the mirror, our considerations encompass a mirror with a width identical to that of the PV panel. Moreover, we have assumed a reflectance value of 0.95 [18] for the mirror's surface properties, taking into account its capacity to efficiently reflect sunlight.

Table 1. Specifications of the Photovoltaic Module (LG NeON 2 LG355N1C-N5)

Parameter	Value
Maximum Power	355 W
Open Circuit Voltage	48.8 V
Short Circuit Current	10.43 A
Cell Efficiency	21.1%
Cell Length	0.156 m
Cell Width	0.156 m
Number of Cells	60 (6 × 10)
Cell type	Monocrystalline
Module Length	1.7 m
Module Width	1.016 m
NOCT	45 °C
Temperature Coefficient	-0.38%/°C
Number of Bypass Diodes	3
Diode Saturation Current	15 A
Series Resistance	0.002 Ω
Parallel Resistance	800 Ω

4.1.1. The boundary conditions

For $L=L_{mus}$ (Maximal useful mirror surface). The Maximal useful surface, is determined with the mirror length L_{mus} (since the mirror width is identical to the panel width in our case), and which is:

$$L_{mus} = l / \tan(h_{min})$$

Since [18]:

$h_{min} = 33.35°$

$l = 1.652$ m

We obtain:

$L_{mus} = 2.51$ m

4.1.2. The optimization function

Equation (2) establishes a relationship between the three parameters targeted for optimization. By utilizing MATLAB, it becomes straightforward to solve this equation in a three-dimensional space. The numerical solutions were visualized as a 3D plot, illustrating the web of possible solutions. This graphical representation facilitates understanding the interplay between the parameters.

To ensure accurate optimization, the interval for each parameter must be clearly defined. The beneficial ranges of the variables were determined as follows:

0<L<Lmus: The mirror length (L) cannot be negative, and beyond Lmus, any additional length offers no practical benefit for flat roofs.

0<α<hmin: The tilt angle (α) must remain positive to suit flat roof installations. Moreover, α>hmin results in undesired shadowing on the PV panel, which must be avoided.

0 <β<70°: A tilt angle (β) beyond 70° shows negligible variation in the 3D graph, making it numerically impractical to consider higher values.

To enhance the clarity of the visualization, additional two-dimensional projections were generated from the 3D graph. These include plots of β=f(α), L=f(β), and L=f(α), alongside the original 3D plot L=f (β, α). These complementary projections, as shown in the figures:

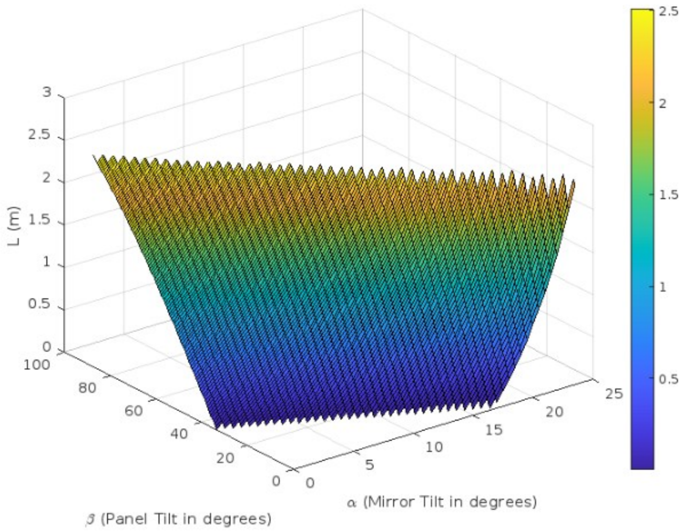

Fig. 11. Variation of Parameters in the 3D Graph

The 3D graph in Fig. 11 (L=f(α,β)) demonstrates a clear interaction between the reflector length (L), the mirror tilt (α), and the panel tilt (β). For lower values of α (closer to horizontal) and higher values of β (steeper tilt), L approaches its maximum value of 2.51 m. The equation confirms this trend, as L is maximized when the numerator, sin(2α+β−hmin), is large, and the denominator sin(hmin−α), remains positive and non-zero. Physically, this means a flatter mirror combined with a steep panel tilt optimizes the sunlight reflected onto the panel.

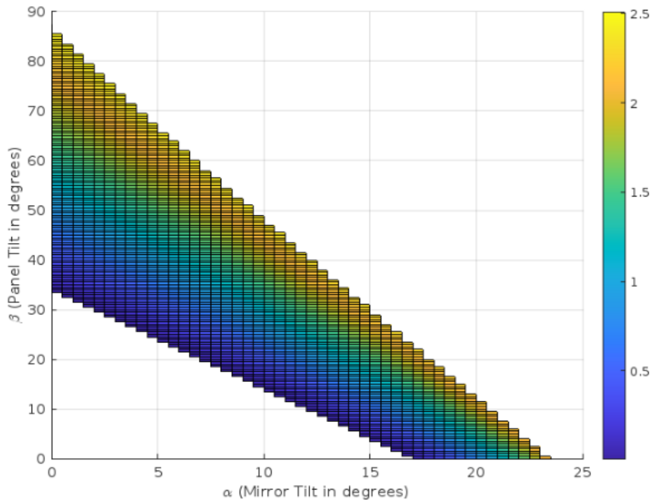

Fig. 12. Dependence of α on β for a Given L

For a fixed L, the relationship between α and β is inversely proportional Fig. 12. As β increases, α must decrease to maintain the same L. This behavior arises because a steeper panel tilt compensates for the reduction in the mirror's contribution. For instance, the graph shows that at high β, α approaches near-horizontal values to sustain an effective reflective geometry.

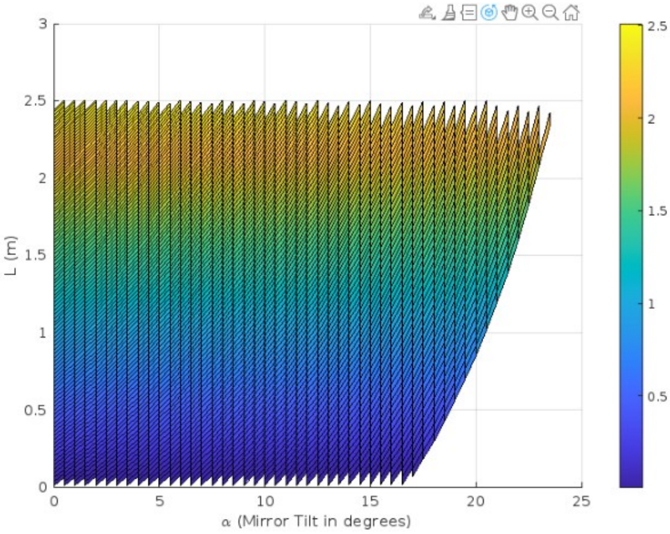

Fig. 13. Dependence of L on β for a Fixed α

At a fixed mirror tilt (α), increasing β leads to a larger reflector length (L) Fig. 13. This behavior is governed by the term (2α+β-hmin) in the numerator of the equation, which grows as β increases. Physically, this implies that a steeper panel tilt improves the alignment of the reflected sunlight with the panel's surface, enhancing the mirror's contribution to solar energy collection. This trend is particularly significant at lower mirror tilt angles, where the system exhibits greater sensitivity to changes in β. The observed increase in L highlights the role of the panel tilt in boosting the overall system efficiency.

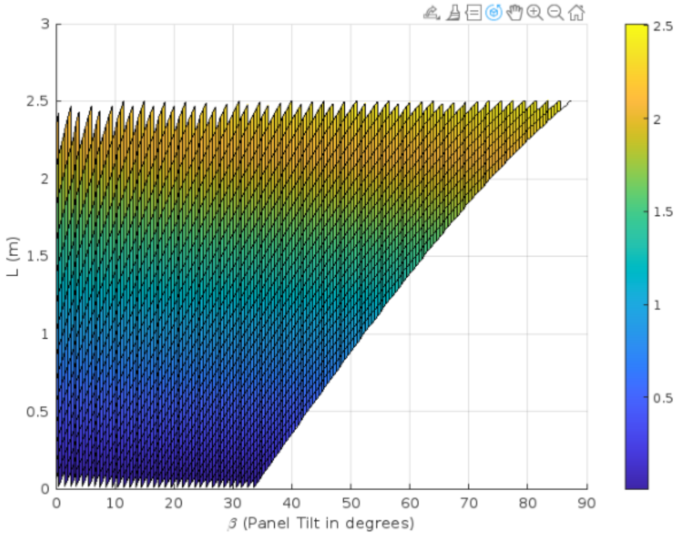

Fig. 14. Dependence of L on α for a Fixed β

When the panel tilt (β) is held constant Fig. 14, increasing the mirror tilt (α) leads to a reduction in the effective reflector length (L). Geometrically, a steeper ground-mounted mirror reduces the reflective contribution to the panel because the sunlight is redirected away from its optimal path toward the panel. The graph clearly shows this behavior, with L decreasing as α increases.

Physically, this suggests that a higher mirror tilt misaligns the reflective surface relative to the sun's rays, decreasing the concentration of sunlight directed toward the PV panel. To maintain effective energy capture, it is essential to keep α low, ensuring that the mirror remains nearly horizontal to maximize its reflective efficiency.

4.1.3. Infinite combinations of solutions

From the Figs. 11, 12, 13, 14: a 3D plot for L=f(α,β) and the three projections β =f(α), L=f(α) and L=f(β). The system's geometry, as represented in the graphs, demonstrates a continuous range of valid combinations of α, β, and L. Geometrically, this flexibility arises from the adaptable interplay between the panel and mirror tilt angles, allowing different configurations to achieve the same reflective effect. For example, increasing β compensates for a reduction in α, or vice versa, to maintain a constant L.

Physically, this adaptability is advantageous in tailoring the system to specific environmental conditions. For instance, during winter, when the sun's elevation angle is lower, a slightly higher α might be required to direct sunlight effectively. Conversely, in summer, minimal α combined with steeper β would optimize solar energy capture. The system's flexibility ensures its feasibility across different locations and seasonal variations.

A. Sensitivity analysis

To understand the influence of each parameter on the performance of the PV system, we performed a sensitivity analysis on the decision variables α, β and L. This step is crucial for identifying which parameters most significantly affect the optimization outcome.

The analysis was carried out by varying one parameter at a time while keeping the others constant, and observing the resulting changes in the reflected sunlight coverage Xref and mirror length L. It was observed that small variations in α had a noticeable impact on the required mirror length, particularly when α approached low values. Similarly, adjusting β affected the system's alignment with both direct and reflected sunlight, confirming the importance of choosing an appropriate tilt angle for seasonal performance. The parameter L was found to be more sensitive at low α values, where reflection geometry becomes more demanding.

This analysis confirms that the system is particularly sensitive to the mirror tilt angle α, which must be carefully adjusted to ensure optimal reflection without causing shading or wasting material.

B. Convergence history

To illustrate the convergence behavior of the optimization function, a numerical procedure was implemented in MATLAB. Starting from an initial estimate of the mirror tilt angle α and the panel tilt angle β, the mirror length L was computed iteratively using Equation (2), which links the geometric parameters of the system. At each step, α and β were progressively adjusted toward their expected optimal values. For every new combination, the corresponding L was recalculated and stored. This simulation was repeated over 30 iterations, providing a sequence of values that reflects how the optimization process stabilizes. The resulting convergence plot presents the evolution of the mirror length L as a function of iteration count:

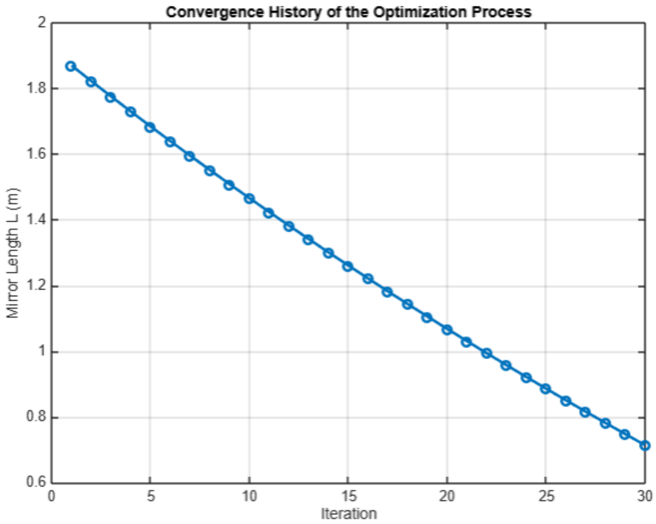

Fig. 15. Convergence history of the optimization process

Fig. 15 illustrates the convergence behavior of the optimization process through the evolution of the mirror length L computed at each iteration using Equation (2). The plot shows a steady and smooth decrease in L, which progressively stabilizes as the number of iterations increases. This indicates that the iterative updates of α and β effectively drive the solution toward a stable configuration. The stabilization of L reflects the algorithm's ability to consistently satisfy the geometric condition defined in Equation (2).

C. Optimizing the system

The optimal configuration of the system lies in balancing the three parameters to maximize energy capture. Geometrically, the graphs indicate that the optimal solution corresponds to minimal α (near-horizontal mirror) and high β (steep panel tilt), resulting in the maximal effective reflector length L. The ground placement of the mirror further underscores the importance of keeping α low, as excessive tilt would limit its contribution to directing sunlight onto the panel.

Physically, the optimal configuration ensures that the panel receives the maximum amount of sunlight reflected from the mirror, enhancing the overall efficiency of the PV system. However, practical considerations such as mechanical stability, wind resistance, and the durability of the ground-mounted mirror must also be considered. Seasonal adjustments to α and β may further optimize performance, particularly in regions with significant variations in solar elevation throughout the year. Computational simulations and optimization algorithms can help refine these parameters while accounting for structural and environmental constraints.

D. Optimization of panel tilt for direct and reflected sunlight

A critical aspect of optimizing the system is ensuring that the panel tilt angle (β) achieves a dual objective: capturing the most beneficial direct sunlight while simultaneously maximizing the beneficial reflections from the mirror. The tilt angle must balance these two contributions to ensure peak energy generation. If β is too steep, the panel may be well-aligned for reflections but lose efficiency in capturing direct sunlight. Conversely, if β is too shallow, the panel might optimize direct sunlight capture but fail to intercept a significant portion of the reflected light. Therefore, the optimization process must carefully evaluate the interplay between direct solar incidence and the angular geometry of the reflected rays, ensuring that the system operates at its highest combined efficiency for the given solar elevation and environmental conditions.

This dual consideration adds complexity to the optimization process but is essential for the overall effectiveness of the mirror-panel system.

4.2. System optimization for the studied location (Khouribga city, Morocco)

In the rest of this research paper, we will focus on our studied location in Khouribga city Morocco, specifically at the Multidisciplinary

Laboratory of Research and Innovation (LaMRI) within the EMAFI Team, Polydisciplinary Faculty, University of Sultan Moulay Slimane. this focus will allow us to have a practical idea, to fix optimal values of β for the panel independently to the mirror, yearly and seasonally, and then optimize more efficiently L and Alpha for each optimal value of β, and even to estimate the power generation based on solar data (solar position, irradiation) and meteorological data for Khouribga city, the choice of the location is also to introduce for the future experimental studies made by the team.

In a recent research paper, Mohamed Nfaoui et al [15], [19] and [20] studied the optimal panel tilt angle for our studied location, the study found that the optimal panel tilt for the region of Khouribga city, considering that the case of this study was a single panel without any reflection instrument mentioned, around 30°, you can also see [21] and [22].

4.2.1. Local optimization based on seasonal panel tilt angles

The study of Mohamed Nfaoui et al also provided a seasonally, and monthly optimal panel tilt, this will allow us to display the different possible optimal combination of the three parameters that we can get flexibly with our method. We will focus in our study on the yearly optimal configuration essentially, and then the seasonally optimal configurations for practical perspectives. Table 2. provides yearly, seasonally optimal panel tilt angles for Khouribga city's location:

Table 2. Yearly, seasonally optimal panel tilt angles for Khouribga city's location [14][15]

Season	β
Autumn	42°
Winter	55°
Spring	19°
Summer	5°
Annual	30°

A. L=f(α) for β=30° in the conditions of our studied location

Now that we know the optimal value of β for a fix system in the studied location, we can plot in two-dimensional space L=f(α), and this will allow us to study, much more specifically, the dependence between the mirror length and its tilt for optimal reflection onto the panel. Fig. 16 shows the plot of L=f(α) for β=30°:

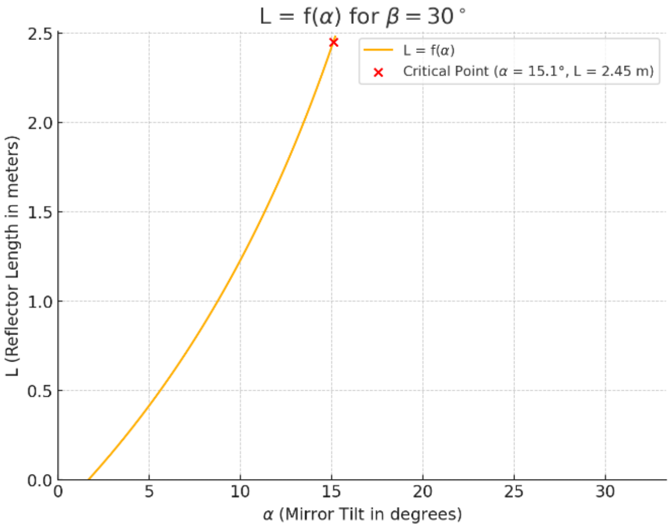

Fig. 16. Variation of L with α for β=30° highlighting the critical point

B. Variation of L with α

From the figure, the length of the mirror L increases nonlinearly with the mirror tilt angle α. At lower values of α, L remains relatively small, as the tilt angle effectively directs sunlight onto the PV panel without requiring an extensive mirror length. However, as α increases, the mirror length grows significantly to maintain the same level of solar reflection. This indicates a geometric dependence on α, where higher angles necessitate larger mirrors to achieve similar levels of incident sunlight on the PV panel.

C. Lower α vs. Higher L: the problematic trade-off

A smaller tilt angle α maximizes the amount of sunlight incident on the mirror because the reflective surface is positioned closer to the direct path of sunlight. However, smaller α values correspond to shorter mirrors, which might limit the total reflected energy onto the panel, especially under varying seasonal solar conditions. Conversely, larger L ensures a higher reflection but comes at the cost of increased material usage and less beneficial α for the incident sunlight to the mirror. This creates a practical challenge: while a larger L might enhance reflection, it risks inefficient resource utilization, making system optimization necessary.

4.2.2. System optimization and the challenge of waste

The primary challenge lies in avoiding the waste of materials while ensuring maximal solar energy capture throughout the year. Mirrors that are too large increase costs and structural complexity without proportionally increasing the system's efficiency. To address this, the critical point is identified where L varies most rapidly with α, representing a point where the mirror length provides significant reflection benefits without excessive material use. The concept of reflection efficiency per unit length (w/cm for L) becomes crucial for quantifying this trade-off and achieving optimal system design.

4.2.3. Mathematical identification of the critical point

The critical point is mathematically determined by analyzing the derivative of L with respect to α. Specifically, the second derivative of L is examined to locate the point where the variation of L accelerates the most. This approach identifies the optimal combination of α and L that balances the competing needs of maximizing reflection and minimizing material waste. Numerically solving for this critical point provides precise values for system design parameters.

4.3. Physical interpretation

The optimization process identified the critical point at α=15.1°, corresponding to L=2.45m. At this point, the mirror length provides a substantial increase in reflection without excessive material use, ensuring consistent performance across seasonal solar variations. Physically, this configuration aligns the mirror tilt angle with the sun's path to maximize the sunlight reflected onto the PV panel, also, the low value of α ensures also better shading conditions in low sun elevations, in the contrary of higher values of α.

Using the same method, we calculated the optimal combinations of the three parameters seasonally for the location of Khouribga city, Table 3. summarizes this optimal value.

Table 3. Optimal values of α, β and L by season

Season	α (°)	β (°)	L (m)
Autumn	11.96	42°	2.51
Winter	8.42	55°	2.51
Spring	15.54	19°	1.6
Summer	18.84	5°	1.1

4.3.1. Temperature effects on lifespan and energy output

When more sunlight is reflected onto a photovoltaic (PV) panel, the increased solar irradiance significantly raises the panel's temperature. This heightened temperature not only reduces the energy conversion efficiency of the PV cells but also accelerates the degradation of materials, shortening the system's lifespan. Temperature-induced losses are particularly critical in regions with high solar irradiance, such as Khouribga, where the reflected energy can intensify these effects. To maximize energy output while ensuring the durability of the PV panels, it is essential to incorporate temperature mitigation strategies in the system design. This highlights the necessity of balancing the benefits of increased solar reflection with the challenges posed by thermal effects to maintain optimal performance over the lifetime of the system.

4.3.2. Comparative analysis of energy yield

As the proposed methodology is fundamentally geometric, it does not include direct energy output simulation. However, to allow a meaningful comparison, we estimated the annual solar energy incident on the PV panel surface under the optimized configuration using clear-sky data for Khouribga. The reflected component was geometrically modeled by accounting for the variation of solar elevation throughout the day and adjusting the reflected footprint (triangular, trapezoidal, or rectangular) accordingly. This estimation assumes ideal reflectivity and stable atmospheric conditions. To validate the approach, the no-mirror case was also estimated under the same assumptions and compared with values reported in the study by Mohamed Nfaoui, who optimized PV tilt angle without reflective surfaces. This cross-reference ensures consistency in evaluating the added benefit of mirror integration.

Table 4. Estimated Annual Energy Yield in Khouribga

Case	Mirror	Energy (kWh/m²/year)
Mohamed Nfaoui (2020)	No	1700
This study (no mirror)	No	1680
This study (with mirror)	Yes	1895

Table 4 shows that the optimized mirror configuration results in a clear gain in captured solar energy, increasing the annual total from 1680 to 1895 kWh/m². This represents a relative improvement of about 12.8% compared to the no-mirror setup. The alignment between our no-mirror estimates and Nfaoui's results also validates the reliability of the method used.

4.3.3. Future research and system optimization

In future research, we aim to develop a comprehensive optimization framework for PV systems by simulating the energy reflected onto the panel and determining the optimal parameters (α and L) for maximum power output by studying P=f (α, L) in the specific context of Khouribga (β=30°). This will involve incorporating the effects of power losses due to temperature increases caused by enhanced solar reflection. To address these losses, we plan to reoptimize the system by integrating efficient and sustainable cooling solutions, such as fins and water-cooling systems powered by micro-eolian turbines and other techniques... These strategies will aim to mitigate overheating while maintaining or enhancing energy production. The ultimate goal of this research is to focus on optimizing existing PV installations, offering practical and cost-effective solutions to improve their performance. By prioritizing the enhancement of current systems over innovations in cell technology or manufacturing processes, this approach ensures the accessibility and sustainability of solar energy optimization for broader applications.

5. Conclusion

This study developed a geometric optimization method to enhance the performance of photovoltaic systems by integrating a reflective mirror on the ground side of the panel. The proposed approach determines the optimal panel tilt angle β = 30°, mirror tilt angle α = 15.1°, and mirror length L = 2.45 m using a mathematical formulation derived from geometric boundary conditions (Equation 2). The optimization ensures minimal shadowing while maximizing reflected irradiance onto the panel throughout useful sunlight hours.

The convergence history and sensitivity analysis confirm the numerical stability of the optimization process and the influence of each design variable. Although the method does not directly calculate harvested energy, an energy estimation was performed using a clear-sky model for Khouribga.

The results show that the optimized mirror configuration can increase the annual irradiance from approximately 1680 kWh/m² (without mirror) to 1895 kWh/m² representing a relative gain of 12.8%. This increase is attributed primarily to additional energy captured during low solar elevation periods in the morning and afternoon.

Compared to the work of Mohamed Nfaoui, which optimized panel tilt without reflectors, this method introduces a complementary dimension that leverages reflected sunlight geometrically. It also offers a straightforward equation that can guide PV-mirror system design without requiring complex simulation tools. These findings demonstrate the potential of geometric optimization for improving solar energy capture, and future studies will focus on validating these results through real irradiance measurements and dynamic simulation.

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