Predicting tensile strength of cemented paste backfill with aid of second order polynomial regression

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Abstract: The materials left behind after the process of separating an ore's valuable fraction from the unprofitable fraction are known as tailings in the mining industry. Mixing tailing, cement and water can create a new material called Cemented paste backfill (CPB). Research and solve the problem of predicting the tensile strength of cement paste backfill based on a polynomial model combined with the Monte Carlo Simulation method. Three models were built to evaluate performance. The optimal performance model is then used to predict the tensile strength of cement paste backfill. The results indicate that using the polynomial regression model has a satisfactory result for predicting the tensile strength of cement paste backfill. The best performance of second order polynomial regression model is evaluated by three metrics such as \( R^2 = 0.958 \), \( \text{RMSE} = 33.211 \text{ kPa} \), \( \text{MAE} = 29.097 \text{ kPa} \) for testing part in predicting the tensile strength of cemented paste backfill. Finally, the influence of Cement/Tailings ratio and Solid content on the tensile strength on tensile strength and importance is also evaluated with aid of the best performance of second order polynomial regression model.

Keywords: Machine learning, cemented paste backfill, polynomial regression, second order.

1. Introduction

In order to maintain a safe working environment, provide a location for the disposal of mill tailings, prevent/minimize surface subsidence from occurring, and provide ground support for the surrounding mine structures, the underground mine openings made during ore production are Cemented Paste Backfill (CPB) as mixing tailing, cement and water using a suitable material [1]. The design and application of various fill media have seen major improvements in the previous few decades, which has led to a rise in the use of paste backfill on a global scale. By outperforming its predecessors, such as hydraulically put backfill, paste fill continues to advance this trend. However, the tensile strength of the cement is extremely important, it helps control cracks and adversely affects the hardness or strength properties [2], [3]. Tensile strength is also related to the action of the shear force on the surface. Tensile strength prediction is a necessary step to enable the optimal selection of materials for construction.

Fall et al. [4] reveal that tailings particle size and density have a significant impact on the performance properties (strength, cost, water demand, and microstructure) of the paste backfill. The tailings particle size, particularly the proportions of fine tailings particles, was shown to have a significant effect on the porosity of the paste backfill and the pore size distribution within it, as well as its water drainage ability and, as a result, its strength development and water requirement for a
given consistency. It was also demonstrated that, in addition to the overall porosity, the pore size distribution has a significant impact on the strength development of the cemented backfill. In the study, Rakine et al. [5] indicate that paste strengthens as cement or solids content and cure time rise. Although there were considerable differences in effective cohesion values for fill mixes, they generally indicated an increase in cement concentration, solids content, and curing time [6].

In addition to empirical studies, many studies apply science and technology to support effective and less expensive solutions to outstanding problems. A significant area of artificial intelligence study is machine learning, which gives machines the capacity to learn and perform certain tasks. People from all industries have been exposed to and intend to employ machine learning as a result of its rising popularity. Algorithms for machine learning are capable of numerous tasks [7]–[9]. Zhang et al. [10] used Talbol gradation theory and neural networks to analyze aggregate gradation in order to obtain the ideal aggregate ratio. The root mean square error (RMSE) of the prediction results for the uniaxial compressive strength (UCS) prediction model that employs the ISTM and incorporates aggregates gradation is 0.0914, the coefficient of determination (R2) is 0.9973, and the variance account for (VAF) is 99.73. The sensitivity analysis of various influencing factors on UCS reveals that all four factors have a substantial effect on UCS, and sensitivity is ranked as follows: The cement content (0.9264)>the slurry concentration (0.9179)>the aggregate gradation (waste rock content) (0.9031)>the curing time (09031). Qi et al. [11] reported in the study the excellent performance of the GBM model achieving a strong positive correlation between the predicted and actual mechanical properties, with R values of 0.963, and 0.887, respectively, 0.886 and 0.899 for the UCS, YS, E, and UTS datasets.

Many reports consider the effects of mechanical walls on uniaxial compressive strength, compressive strength. However, there is no research on tensile strength, which is also an important factor affecting the strength of materials. The study is a complement to the studies of the tensile strength of cement paste backfill gradually becoming complete. Evaluate the reliability of the proposed model and analyze the influence of the components on the tensile strength. At the same time, the study is also interested in the influence of the components.

2. Machine learning approach

2.1. Description of database

The data used in the study included 77 samples for the Uniaxial Tensile Strength (UTS) test used in the previous study by Qi et al. [12]. The data includes 8 input variables: Gs, D10, D50, Cu, Cc, Cement/Tailings, Solid content, and the output variable Tensile Strength (TS). Where, Gs is the specific gravity of the cemented paste backfill, D10 and D50 are the diameter of grain size (mm) permits only 10% and 50% grain of tailing passing, respectively, Cu is the uniform coefficient of tailing, Cc is coefficient of curvature of tailing, Cement/Tailing is the ratio of used cement and tailing in CPB, and solid content is the percentage of mix "cement and tailing" in CPB. To ensure that the model accurately predicts the tensile strength of the cement paste backfill, the input variables include different contents.

The correlation between the input variables is shown by the correlation matrix in Figure 1. Tensile strength and cement/tailing ratio have a higher correlation than other factors with r=0.76. Cu is more explanatory than Cc and Solid content, based on their correlation coefficients, r=0.20 for Cu and r=0.09 for Cc and Solid content. Observing Figure 1, it can be seen that, Gs has almost no statistically significant correlation with most of the observed parameters. The measured correlation coefficients show a strong correlation between the parameters. There is a statistically significant correlation between Gs and Cc with r=0.43. A similar correlation is also found on other parameter
pairs such as between Cu and Cc with $r=0.56$, D10 and D50 with $r=0.98$. While other input parameters have a relatively negative influence on the cement/tailing ratio, solid content has a particularly large positive influence on it. It can be seen that the cement/tailing ratio has the most influence on tensile strength and solid content has an important effect on cement.

Figure 1. Simple analysis of database including the histogram of each feature and correlation between input and output variable

2.2. Polynomial regression-linear regression algorithm

Polynomial regression is a form of regression model or analysis in which the relationship between x and y variables is independent and dependent variables are modeled as nth degree of the polynomial. Polynomial regression speaks to the fact that there is a polynomial relationship between predictors and response variables regardless of the number of features. Linear regression is linear in the parameters, not the variables. It is possible to perform any transformation from them and still have a linear model. Therefore, polynomial regression is a special case of linear regression. Polynomial regression is like multiple regression, when performing polynomial regression just doing a multiple regression with multiple transformations of a single variable. Thus, the algorithm tells if a single
term is statistically significant also if a variable is significant.

The polynomial equation is not unique, the following is the most conventional polynomial regression

\[ h_y(x) = \theta_0 + \theta_1 x + \theta_2 x^2 + \ldots + \theta_n x^n \]  

Treat \( x, x^2, \ldots, x^n \) as \( n \) variables. This is a linear function of \( \theta_0, \theta_1, \theta_2, \ldots, \theta_n \)

Cost function

\[ L(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{2n} \sum_{i=1}^{n} (h_y(x^i) - y^i)^2 \]  

2.3. Performance metrics of model evaluation

Three metrics were utilized in this work to assess the correctness of the created model, namely correlation coefficient (\( R^2 \)), RMSE (Root Mean Squared Error), and MAE (Mean Absolute Error)

The rate of variation of the dependent variable as a result of the combined variation of the explanatory factors is represented by \( R^2 \). The predicted value is nearer to the goal value, and \( R^2 \) is closer to 1. The MAE statistic measures the average number of forecasting errors without taking into account the direction of the errors. The discrepancy between values predicted by a model or estimator and the values observed is commonly measured using the root mean square error (RMSE). Given that its interpretation in terms of relative inaccuracy is incredibly evident. The mean model prediction error per unit of the desired output is a characteristic of both the MAE and RMSE criterion. The better the RMSE and MAR values are compared to higher R2 scores.

\[ R^2 = \frac{\sum_{k=1}^{N}(val^{ex}_k - val^{avg}_k)^2 - \sum_{k=1}^{N}(val^{ex}_k - val^{pre}_k)^2}{\sum_{k=1}^{N}(val^{ex}_k - val^{avg}_k)^2} \]  

\[ RMSE = \frac{1}{N} \sqrt{\sum_{k=1}^{N}(val^{ex}_k - val^{pre}_k)^2} \]

\[ MAE = \frac{1}{N} \sum_{k=1}^{N}|val^{ex}_k - val^{pre}_k| \]  

Where, \( N \) is the number of datasets, \( val^{ex} \) and \( val^{avg} \) are the experimental value and mean experimental value, respectively. \( val^{pre} \) is the predicted value by ML model.

3. Result and discussion

3.1. Evaluating performance of polynomial regression model

The three models PR1, PR2 and PR3 were evaluated with data repeated 1000 times through Monte-Carlo Simulation (MCS) randomization. To verify the performance of different polynomial models for predicting tensile strength, the data is divided into two parts as training sample data and test data. In Figure 2, different colors are used to distinguish between the training data and the test data of the polynomials. The PR1 model showed lower performance than the PR2 and PR3 models. The expected difference in performance indicators of the PR2 and PR3 models is relatively small when looking at Figure 2. For more insight, the values of the performance indicators for the models are shown. shown in Table 1 for the training dataset and Table 2 for the testing dataset. According to table 1, model PR2 has a smaller average \( R^2 \) value than PR3 (0.962<0.965), however, two values of RMSE (34.156) and MAE (32.686) on average of PR2 have more optimal value than PR3 model. with model PR3 with the average RMSE, MAE values are 32.686 and 25.024 respectively. In addition, it can be seen that the standard deviation of the two polynomial models on the training data set is not too large, almost equivalent. Surprisingly, the observed performance on the test data set, model PR2 has the average \( R^2(0.898) \) value, which is larger than the average \( R^2(0.875) \) value of the PR3 model, the RMSE (48.222) and average MAE (37.788) of PR2 model have smaller values than RMSE=54.588, and average MAE=39.926 of PR3 model. In addition, the standard deviation of
the PR3 model is much larger than the standard deviation of the PR2 model. Therefore, it can be said that the PR2 model gives a more reliable performance than the PR1 and PR3 models.
Figure 2. Performance metrics R2, RMSE and MAE of 3 order PR models for training and testing part

Table 1. Performance values of three polynomial regression model for the training dataset with 1000 random MCS simulations

<table>
<thead>
<tr>
<th></th>
<th>PR1</th>
<th>PR2</th>
<th>PR3</th>
<th>PR1</th>
<th>PR2</th>
<th>PR3</th>
<th>PR1</th>
<th>PR2</th>
<th>PR3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.742</td>
<td>0.905</td>
<td>0.905</td>
<td>43.157</td>
<td>21.634</td>
<td>20.767</td>
<td>32.505</td>
<td>16.626</td>
<td>15.721</td>
</tr>
<tr>
<td>Max</td>
<td>0.910</td>
<td>0.982</td>
<td>0.983</td>
<td>87.390</td>
<td>40.459</td>
<td>49.916</td>
<td>66.161</td>
<td>34.196</td>
<td>36.208</td>
</tr>
<tr>
<td>Average</td>
<td>0.828</td>
<td>0.962</td>
<td>0.965</td>
<td>73.231</td>
<td>34.156</td>
<td>32.686</td>
<td>53.262</td>
<td>27.131</td>
<td>25.024</td>
</tr>
<tr>
<td>Std</td>
<td>0.023</td>
<td>0.011</td>
<td>0.011</td>
<td>7.545</td>
<td>2.449</td>
<td>2.976</td>
<td>5.531</td>
<td>2.233</td>
<td>2.480</td>
</tr>
</tbody>
</table>

Table 2. Performance values of three polynomial regression model for the testing dataset with 1000 random MCS simulations

<table>
<thead>
<tr>
<th></th>
<th>PR1</th>
<th>PR2</th>
<th>PR3</th>
<th>PR1</th>
<th>PR2</th>
<th>PR3</th>
<th>PR1</th>
<th>PR2</th>
<th>PR3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.401</td>
<td>0.570</td>
<td>0.417</td>
<td>39.069</td>
<td>28.391</td>
<td>28.757</td>
<td>32.234</td>
<td>21.789</td>
<td>21.063</td>
</tr>
<tr>
<td>Max</td>
<td>0.932</td>
<td>0.981</td>
<td>0.976</td>
<td>141.113</td>
<td>103.522</td>
<td>179.065</td>
<td>91.571</td>
<td>67.653</td>
<td>105.265</td>
</tr>
<tr>
<td>Average</td>
<td>0.747</td>
<td>0.898</td>
<td>0.875</td>
<td>83.140</td>
<td>48.222</td>
<td>54.588</td>
<td>59.914</td>
<td>37.788</td>
<td>39.926</td>
</tr>
<tr>
<td>Std</td>
<td>0.103</td>
<td>0.068</td>
<td>0.094</td>
<td>16.912</td>
<td>8.285</td>
<td>15.670</td>
<td>9.877</td>
<td>5.798</td>
<td>8.630</td>
</tr>
</tbody>
</table>

3.2. Predicting tensile strength of cement paste backfill

The model’s generalization performance is shown in Figure 3. The red dashed line represents the experimental value, the black solid line represents the predicted value. It can be seen that the prediction results are relatively close to the experimental value, in about 20 samples, the prediction results are almost linear compared to the experimental value. To take a closer look at the prediction accuracy, a statistic of the change in error for the value of tensile strength of cement paste backfill is shown in Figure 4.
Figure 3. Comparing predicted tensile strength with true tensile strength for (a) training dataset and (b) testing dataset

Figure 4. Error of predicted tensile strength by the polynomial regression
In this study, the relationship between the dependent variable tensile strength and the independent variables Gs, D10, D50, Cu, Cc, Cement/Tailings, Solid content can be described by a matrix.

Tensile strength (kPa) = $\mathbf{A} \cdot \mathbf{B} + 3343.14$

In there:

$\mathbf{A} = \begin{bmatrix} 1.04 & -0.01 & 0.19 & -33.52 \\ 9.61 & -3189.52 & -5.55 & 0.24 \\ 2.09 & -86.03 & 28.12 & -7133.64 \\ -10.73 & -0.43 & -0.15 & 0.31 \\ -864.96 & -1.99 & 2.66 & 2.76 \\ -4325.35 & -10.83 & 59.55 & 328.39 \\ 2.23 & -1615.73 & -5.49 & 344.78 \end{bmatrix}$

$\mathbf{B} = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 \\ X_5 & X_6 & X_7 & X_1X_2 \\ X_1X_3 & X_1X_4 & X_1X_5 & X_1X_6 \\ X_1X_7 & X_2X_3 & X_2X_4 & X_2X_5 \\ X_2X_4 & X_2X_5 & X_2X_6 & X_2X_7 \\ X_3X_6 & X_3X_7 & X_4X_5 & X_4X_6 \\ X_4X_7 & X_5X_6 & X_5X_7 & X_6X_7 \end{bmatrix}$

Table 3. Rank value validation for the polynomial formula in predicting tensile strength of cement paste backfill

<table>
<thead>
<tr>
<th>Unit</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gs</td>
<td>2.08</td>
<td>2.83</td>
</tr>
<tr>
<td>D10</td>
<td>0.005</td>
<td>0.248</td>
</tr>
<tr>
<td>D50</td>
<td>0.049</td>
<td>1.442</td>
</tr>
<tr>
<td>Cu</td>
<td>4.65</td>
<td>12.7</td>
</tr>
<tr>
<td>Cc</td>
<td>0.9</td>
<td>2.04</td>
</tr>
<tr>
<td>Cement/Tailings</td>
<td>0.05</td>
<td>0.25</td>
</tr>
<tr>
<td>Solid content</td>
<td>68</td>
<td>80</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>10</td>
<td>920</td>
</tr>
</tbody>
</table>

In the test, tensile strength, solid content, and cement/tailings are tested respectively to see the relationship between them Figure 5. As the illustration shows, for each different cement/tailing ratio, based on the increase of the solid content increases, the extent of tensile strength development is also markedly different. For the ratio cement/tailings=0.22, a rapid increase in tensile strength can be observed. With a solid content of 68%, the tensile strength obtained is the lowest of the four cases. However, as the solid content increases, the tensile strength value also increases rapidly, becoming the case with the highest tensile strength of the four cases. The orange graph represents the case of cement/tailings=0.10. When the solid content=68%, this is the case with the highest tensile strength value. However, as the solid content increases, the development of the tensile strain of this case is relatively slow.

Figure 5. Influence of Cement/Tailings ratio and Solid content on the tensile strength with aide of the polynomial regression model

4. Conclusions

In this paper, a polynomial model is applied to predict the tensile strength of cement paste backfill. The study evaluated the reliability of the integrated polynomial regression model Monte Carlo Simulation for predicting the tensile strength of cement paste backfill. The data used in the study included 77 samples of the UTS test, the MCS technique was used to replicate 1000 times. By comparing the performance between polynomial models...
models, the model with the best performance is selected for predicting tensile strength. The results show that, using the multinomial model has a significant effect with the value of $R^2=0.958$, RMSE=33.211kPa, and MAE=29.097kPa. It can be seen that this model has the potential to predict the tensile strength of cement paste backfill with small deviation and the calculation procedure is feasible and simple. The relationship between the cement/tailing ratio and solid content is also considered. With this new discovery, the selection of the content of component in CPB can be reasonably adjusted to the needs of the user.

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References


