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Nonlinear thermal stability of plates with auxetic FG-CNTRC core and FG-CNTRC face sheets considering temperature-dependent material properties

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Abstract: This study examines the nonlinear thermal stability of functionally graded carbon nanotube-reinforced composite (FG-CNTRC) sandwich plates incorporating an auxetic core and resting on a two-parameter Pasternak elastic foundation using an analytical framework. The material properties of the FG-CNTRC constituents are assumed to be temperature-dependent, and the structural response is evaluated under a uniform temperature rise. The auxetic core is characterized through a homogenization scheme in which its effective properties are derived from FG-CNTRC cell walls with carbon nanotubes aligned along the re-entrant members. This modeling strategy enables the directional reinforcement effect of CNTs to be consistently transferred to the equivalent orthotropic behavior of the auxetic core. A higher-order shear deformation theory, combined with von Kármán geometric nonlinearity is employed to describe the kinematics of the sandwich plates. By applying an energy-based formulation, the nonlinear governing equations are expressed and subsequently solved by Ritz method with admissible trigonometric functions. Closed-form expressions for the critical thermal buckling temperature are obtained, and the nonlinear postbuckling equilibrium paths are determined. The comprehensive investigations are conducted to assess the influences of CNT volume fraction, grading patterns, auxetic geometric parameters, foundation, and temperature-dependent effects on both the nonlinear stability responses.

Keywords: Functionally graded carbon nanotube-reinforced composite, Nonlinear thermal buckling and postbuckling, Higher-order shear deformation theory, Auxetic core, Pasternak elastic foundation, Temperature-dependent material properties, Plate.

1. Introduction

Plate-like structural components are widely employed in engineering applications due to their efficient load-carrying capability and structural

versatility. These elements are commonly characterized by flat or mildly curved geometries with simple shapes such as rectangular or circular configurations. Owing to their favorable stiffness-

to-weight ratio, plate and panel structures are capable of distributing external loads effectively while minimizing material consumption. As a result, they are extensively utilized in various fields, including civil engineering (e.g., floor systems and wall elements), mechanical systems (e.g., protective casings), and transportation infrastructure (e.g., bridge decks and tunnel linings). Functionally graded materials (FGMs) are advanced composites characterized by a continuous variation in composition and microstructure through their thickness, resulting in smooth transitions of mechanical and thermal properties. Such gradation enables FGMs to achieve improved performance in terms of strength, thermal resistance, and durability, making them highly attractive for applications in aerospace, energy systems, and mechanical engineering. Extensive researches have been conducted on the mechanical and thermal behavior of FGM plates using various theoretical and numerical approaches. For instance, linear and nonlinear responses, including mechanical and thermal buckling as well as dynamic behavior, have been analyzed using first-order shear deformation theory (FSDT) and element-free and meshless local Petrov-Galerkin methods [1,2]. Higher-order shear deformation theory (HSDT), combined with perturbation techniques, has also been widely applied to capture nonlinear behavior of FGM structures with improved accuracy [3]. Moreover, the nonlinear thermo-mechanical stability of stiffened FGM plates has been investigated using enhanced smeared stiffener models together with Galerkin-based formulations within both the FSDT and HSDT frameworks [4,5]. The influence of thermal loading and foundation interaction on the bending behavior of FGM cylindrical panels has been addressed by incorporating HSDT and perturbation methods [6]. In addition, advanced formulations considering the physical neutral surface have been utilized to study large-amplitude vibration characteristics of FGM

cylindrical panels [7], while state-space and three-dimensional elasticity approaches have been adopted to examine their thermo-elastic responses in greater detail [8,9]. The nonlinear buckling behavior of doubly curved FGM panels has also been reported using higher-order theories coupled with perturbation solutions [10].

Functionally graded carbon nanotube-reinforced composites (FG-CNTRCs) represent a class of advanced materials in which carbon nanotubes are incorporated into a matrix with a spatially varying distribution across the thickness. This gradation, combined with the exceptional mechanical and multifunctional properties of CNTs, leads to significant enhancements in stiffness, strength, and thermal and electrical conductivities. Due to these superior characteristics, FG-CNTRCs have attracted considerable attention in high-performance applications such as aerospace structures, automotive components, and advanced engineering systems. The structural responses of FG-CNTRC plates have been extensively investigated in the literature using various analytical and numerical techniques. Nonlinear thermal buckling responses of FG-CNTRC rectangular plates have been examined based on higher-order shear deformation theories in combination with perturbation and multiscale approaches [11,12]. In addition, different solution strategies, including meshfree Ritz formulations, two-step perturbation schemes, and Galerkin-based methods, have been employed to analyze the buckling, postbuckling, and dynamic nonlinear stability of FG-CNTRC plate and shell structures with various geometries, such as skew plates and cylindrical panels [13-16]. Furthermore, the dynamic characteristics of FG-CNTRC and sandwich FG-CNTRC structures, including free vibration, forced vibration, and wave propagation, have been widely explored under diverse conditions. These studies have considered effects such as arbitrary geometries, interactions with elastic foundations, integration with smart sensing

and actuation systems, and emerging applications involving multifunctional structures [17-20]. The sandwich FG-CNTRC plates were considered in studies of nonlinear buckling behavior, and the effects of a multilayer corrugated core were taken into account [21]. In the nonlinear vibration problems, the doubly curved FG-CNTRC shell stiffened by eccentric FG-CNTRC stiffeners was also mentioned using a semi-analytical approach [22].

Auxetic material, with negative Poisson's ratio, exhibits the unusual mechanical behavior of lateral expansion when subjected to tensile loading. This counterintuitive deformation mechanism endows such materials with enhanced properties, including improved energy absorption, shear resistance, and indentation resilience. Owing to these advantages, auxetic structures have attracted increasing attention in a wide range of engineering applications. In particular, auxetic configurations are frequently employed in structures exposed to dynamic and impact loading conditions, where their ability to undergo stable deformation and redistribute stresses becomes highly beneficial. Numerous studies have demonstrated the significant influence of auxetic cores on the mechanical performance of sandwich structures, highlighting their effectiveness in improving stiffness, stability, and overall structural integrity under complex loading environments [23-29].

Despite the extensive research on FG-CNTRC structures and auxetic materials, most existing studies have primarily focused on configurations in which the auxetic core is made of conventional isotropic or weakly anisotropic materials. In such cases, the intrinsic directional characteristics of fiber-like reinforcements, such as carbon nanotubes, have not been fully exploited in the design of auxetic architectures. Moreover, although nonlinear thermal buckling and postbuckling behaviors of FG-CNTRC plates have been widely investigated, limited attention has

been given to sandwich structures in which both the face sheets and the auxetic core are composed of functionally graded CNT-reinforced materials. In addition, the influences of temperature-dependent material property on the nonlinear thermal buckling of such advanced sandwich systems remains insufficiently explored, particularly when combined with foundation interaction effects. The coupling between material gradation, CNT orientation, auxetic geometry, and thermal loading introduces a complex multi-parameter problem that has not been comprehensively addressed in the literature.

Motivated by these limitations, the present study aims to develop an analytical model for the nonlinear thermal buckling and postbuckling behavior of FG-CNTRC sandwich plates with an auxetic core. The auxetic core is constructed from FG-CNTRC with CNTs aligned along the cell walls, allowing the anisotropic reinforcement effect to be incorporated into the homogenized properties. The formulation further accounts for temperature-dependent material properties and the interaction with a two-parameter Pasternak elastic foundation. The obtained results provide new insights into the role of CNT alignment and auxetic configurations in enhancing the thermal stability of advanced composite sandwich structures.

2. Material modelling of FG-CNTRC sandwich plates with auxetic core

In the present paper, a sandwich plate configuration consisting of an auxetic core and FG-CNTRC face sheets is considered. The geometrical characteristics of the plate, including the auxetic core and the FG-CNTRC layers, are illustrated in Fig. 1. To facilitate practical manufacturability, the auxetic core is assumed to be constructed from FG-CNTRC material, ensuring material compatibility with the face sheets.

The volume fraction of CNTs in the outer face sheets, corresponding to the region $\left(-\frac{h}{2} \leq z \leq -\frac{h_{AU}}{2}\right)$, can be considered to vary

through plate's thickness respecting to linear functions, as

- UD type: $\bar{V}_{CNT} = V_{CNT}^*$, (1a)

- FG-X type: $\bar{V}_{CNT} = \left| \frac{8z + 2h + 2h_{AU}}{h - h_{AU}} \right| V_{CNT}^*$, (1b)

- FG-O type: $\bar{V}_{CNT} = \left(2 - \left| \frac{8z + 2h + 2h_{AU}}{h - h_{AU}} \right| \right) V_{CNT}^*$. (1c)

The CNT volume fraction within the auxetic core, defined over the region $\left(-\frac{h_{AU}}{2} \leq z \leq \frac{h_{AU}}{2} \right)$, can be considered to vary through plate's thickness

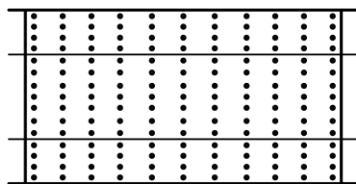
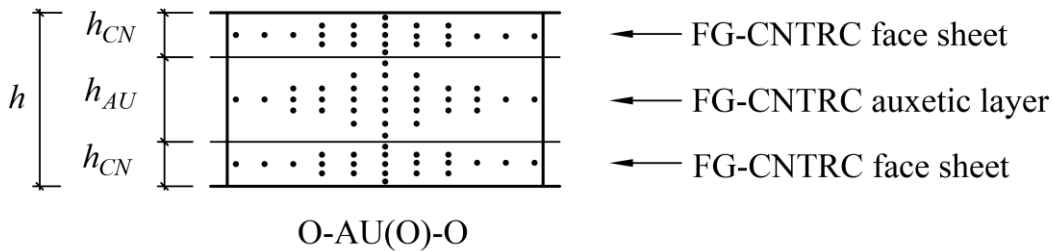
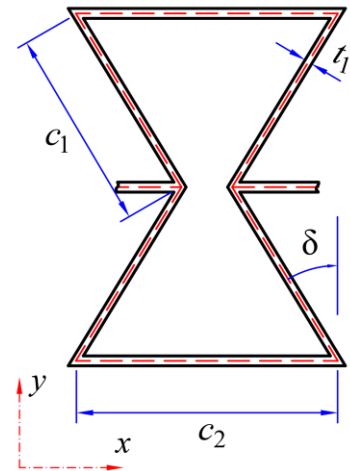
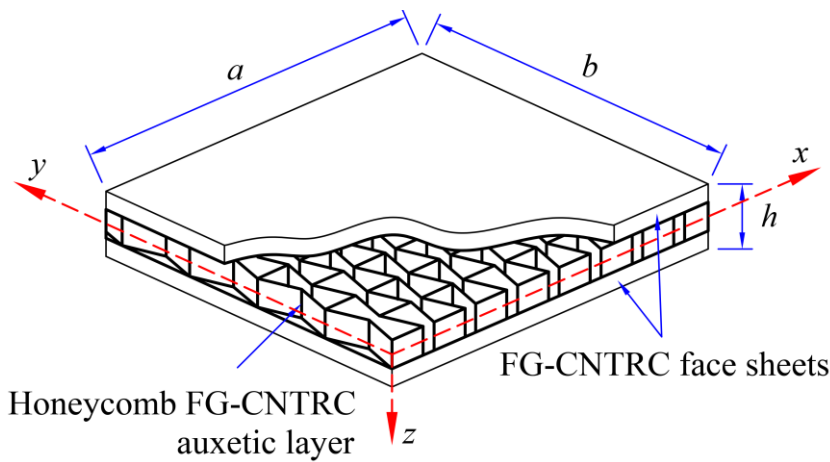
respecting to linear functions, as

- UD type: $\bar{V}_{CNT} = V_{CNT}^*$, (2a)

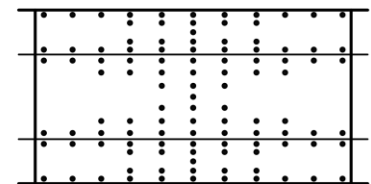
- FG-X type: $\bar{V}_{CNT} = \frac{4|z|}{h_{AU}} V_{CNT}^*$, (2b)

- FG-O type: $\bar{V}_{CNT} = \left(2 - \frac{4|z|}{h_{AU}} \right) V_{CNT}^*$. (2c)

The CNT volume fraction in the inner face sheet, corresponding to the region $\left(\frac{h_{AU}}{2} \leq z \leq \frac{h}{2} \right)$, can be considered to vary through plate's thickness respecting to linear functions, as



UD-AU(UD)-UD



X-AU(X)-X

Fig. 1. Geometric parameters of the FG-CNTRC sandwich plates, auxetic core and CNT distribution patterns through the thickness.

- UD type: $\bar{V}_{CNT} = V_{CNT}^*$, (3a)

- FG-X type: $\bar{V}_{CNT} = \left| \frac{8z - 2h - 2h_{AU}}{h - h_{AU}} \right| V_{CNT}^*$, (3b)

- FG-O type: $\bar{V}_{CNT} = \left(2 - \left| \frac{8z - 2h - 2h_{AU}}{h - h_{AU}} \right| \right) V_{CNT}^*$. (3c)

The face sheets and the auxetic core are

configured based on three distinct CNT distribution schemes, namely X-AU(X)-X and O-AU(O)-O, and UD-AU(UD)-UD.

The effective elastic properties of the FG-CNTRC material are evaluated using the extended rule of mixtures. Accordingly, the expressions for the Young and shear moduli, and Poisson's ratio are obtained as follows [11]

$$\begin{aligned}\bar{E}_{11}^{\text{CNT}} &= \bar{\eta}_1^{\text{CNT}} \bar{V}_{\text{CNT}} \bar{E}_{11}^{\text{CN}} + \bar{V}_M \bar{E}_M, \\ \frac{\bar{\eta}_2^{\text{CNT}}}{\bar{E}_{22}^{\text{CNT}}} &= \frac{\bar{V}_{\text{CNT}} \bar{E}_M + \bar{V}_M \bar{E}_{22}^{\text{CN}}}{\bar{E}_{22}^{\text{CN}} \bar{E}_M}, \\ \frac{\bar{\eta}_3^{\text{CNT}}}{\bar{G}_{12}^{\text{CNT}}} &= \frac{\bar{V}_{\text{CNT}} \bar{G}_M + \bar{V}_M \bar{G}_{12}^{\text{CN}}}{\bar{G}_{12}^{\text{CN}} \bar{G}_M}, \\ \bar{V}_{12}^{\text{CNT}} &= V_{\text{CNT}}^* \bar{V}_{12}^{\text{CN}} + \bar{V}_M \bar{V}_M,\end{aligned}\quad (4)$$

where $\bar{V}_{\text{CNT}}$ and $\bar{V}_M$ denote the volume fractions of carbon nanotubes and the matrix material, respectively, satisfying the condition ($\bar{V}_{\text{CNT}} + \bar{V}_M = 1$). The parameters $\bar{\eta}_j$ ($j=1, 2, 3$) are efficiency coefficients introduced to account for nanoscale

$$\begin{aligned}E_{11}^{\text{AU}} &= \frac{E_{\text{CNT}}^{\text{AU}} \vartheta_3^3 (\vartheta_1 - \sin \delta)}{\cos^3 \delta \left[1 + (\tan^2 \delta + \vartheta_1 \sec^2 \delta) \vartheta_3^2 \right]}, E_{22}^{\text{AU}} = \frac{E_{\text{CNT}}^{\text{AU}} \vartheta_3^3}{\cos \delta (\vartheta_1 - \sin \delta) (\tan^2 \delta + \vartheta_3^2)}, \\ G_{12}^{\text{AU}} &= \frac{E_{\text{CNT}}^{\text{AU}} \vartheta_3^3}{\vartheta_1 (1 + 2\vartheta_1) \cos \delta}, G_{23}^{\text{AU}} = \frac{\vartheta_3 \cos \delta}{\vartheta_1 - \sin \delta} \frac{E_{\text{CNT}}^{\text{AU}}}{2(1 + v_{\text{CNT}}^{\text{AU}})}, \\ G_{13}^{\text{AU}} &= \frac{E_{\text{CNT}}^{\text{AU}} \vartheta_3}{4(1 + v_{\text{CNT}}^{\text{AU}}) \cos \delta} \left[\frac{\vartheta_1 - \sin \delta}{1 + 2\vartheta_1} + \frac{\vartheta_1 + 2 \sin^2 \delta}{2(\vartheta_1 - \sin \delta)} \right], \\ v_{21}^{\text{AU}} &= -\frac{\sin \delta (1 - \vartheta_3^2)}{(\tan^2 \delta + \vartheta_3^2) (\vartheta_1 - \sin \delta)}, v_{12}^{\text{AU}} = -\frac{\sin \delta (1 - \vartheta_3^2) (\vartheta_1 - \sin \delta)}{\left[1 + (\vartheta_1 \sec^2 \delta + \tan^2 \delta) \vartheta_3^2 \right] \cos^2 \delta}, \alpha_{\text{AU}} = \alpha_{\text{CNT}}^{\text{AU}}.\end{aligned}\quad (6)$$

where the effective properties of the FG-CNTRC auxetic core can be represented by thickness-averaged material parameters obtained through homogenization, as

$$\begin{aligned}\vartheta_1 &= c_2/c_1, \vartheta_3 = t_1/c_1, \\ E_{\text{CNT}}^{\text{AU}} &= \frac{1}{h_{\text{AU}}} \int_{-h_{\text{AU}}/2}^{h_{\text{AU}}/2} \bar{E}_{11}^{\text{CNT}}(z) dz, \\ v_{\text{CNT}}^{\text{AU}} &= \frac{1}{h_{\text{AU}}} \int_{-h_{\text{AU}}/2}^{h_{\text{AU}}/2} \bar{v}_{12}^{\text{CNT}}(z) dz,\end{aligned}$$

effects. The elastic properties of the CNTs are represented by $\bar{E}_{11}^{\text{CN}}$, $\bar{E}_{22}^{\text{CN}}$, and $\bar{G}_{12}^{\text{CN}}$, while $\bar{E}_M$ and $\bar{G}_M$ correspond to those of the isotropic matrix. Furthermore, the Poisson's ratios $\bar{v}_{12}^{\text{CN}}$ and $\bar{v}_M$ are respectively defined for the CNTs and the matrix.

The coefficients of thermal expansion of FG-CNTRC in the x - and y -directions are expressed as follows.

$$\begin{aligned}\bar{\alpha}_{11}^{\text{CNT}} &= \bar{V}_{\text{CNT}} \bar{\alpha}_{11}^{\text{CN}} + \bar{V}_M \bar{\alpha}_M, \\ \bar{\alpha}_{22}^{\text{CNT}} &= (1 + \bar{v}_{12}^{\text{CN}}) \bar{V}_{\text{CNT}} \bar{\alpha}_{22}^{\text{CN}} + (1 + \bar{v}_M) \bar{V}_M \bar{\alpha}_M - \bar{V}_{12}^{\text{CNT}} \bar{\alpha}_{11}^{\text{CNT}},\end{aligned}\quad (5)$$

where the thermal expansion coefficients $\bar{\alpha}_{11}^{\text{CN}}$, $\bar{\alpha}_{22}^{\text{CN}}$, and $\bar{\alpha}_M$ are respectively defined for the CNTs and the isotropic matrix.

The effective elastic moduli of the honeycomb auxetic core are obtained through a homogenization approach [23], as expressed below

$$\alpha_{\text{CNT}}^{\text{AU}} = \frac{1}{h_{\text{AU}}} \int_{-h_{\text{AU}}/2}^{h_{\text{AU}}/2} \bar{\alpha}_{11}^{\text{CNT}}(z) dz.$$

The thermo-elastic constitutive equations for the FG-CNTRC sandwich plate are formulated as follows

$$\begin{aligned}\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} &= \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \hat{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x - \bar{\alpha}_{11}^{\text{CNT}} \Delta T \\ \varepsilon_y - \bar{\alpha}_{22}^{\text{CNT}} \Delta T \\ \gamma_{xy} \end{bmatrix}, \\ \sigma_{xz} &= \bar{Q}_{44} \gamma_{xz}, \quad \sigma_{yz} = \bar{Q}_{55} \gamma_{yz},\end{aligned}\quad (7)$$

where ΔT denotes the temperature change from the initial stress-free state to the deformed configuration of the plate, and the reduced stiffness coefficients $\bar{Q}_{ij}$ for the face sheets can be determined accordingly.

$$\bar{Q}_{11} = \frac{\bar{E}_{11}^{CNT}}{1 - \bar{v}_{12}^{CNT} \bar{v}_{21}^{CNT}}, \quad \bar{Q}_{22} = \frac{\bar{E}_{22}^{CNT}}{1 - \bar{v}_{12}^{CNT} \bar{v}_{21}^{CNT}},$$

$$\bar{Q}_{12} = \frac{\bar{v}_{21}^{CNT} \bar{E}_{11}^{CNT}}{1 - \bar{v}_{12}^{CNT} \bar{v}_{21}^{CNT}}, \quad \bar{Q}_{44} = \frac{\bar{Q}_{55}}{1.2} = \bar{Q}_{66} = \bar{G}_{12}^{CNT}.$$

for auxetic-core

$$\bar{Q}_{11} = \frac{E_{11}^{AU}}{1 - v_{12}^{AU} v_{21}^{AU}}, \quad \bar{Q}_{22} = \frac{E_{22}^{AU}}{1 - v_{12}^{AU} v_{21}^{AU}},$$

$$\bar{Q}_{12} = \frac{v_{21}^{AU} E_{11}^{AU}}{1 - v_{12}^{AU} v_{21}^{AU}},$$

$$\bar{Q}_{44} = G_{13}^{AU}, \quad \bar{Q}_{55} = 1.2 G_{23}^{AU}, \quad \bar{Q}_{66} = G_{12}^{AU}.$$

3. Theoretical formulation of FG-CNTRC sandwich plates

Based on the HSDT and incorporating von Kármán-type geometric nonlinearity, the strain components at a distance z from the mid-surface of the plate are expressed as [11]

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \hat{\varepsilon}_x^{[0]} \\ \hat{\varepsilon}_y^{[0]} \\ \hat{\gamma}_{xy}^{[0]} \end{Bmatrix} + z \begin{Bmatrix} \hat{\varepsilon}_x^{[1]} \\ \hat{\varepsilon}_y^{[1]} \\ \hat{\gamma}_{xy}^{[1]} \end{Bmatrix} + z^3 \begin{Bmatrix} \hat{\varepsilon}_x^{[3]} \\ \hat{\varepsilon}_y^{[3]} \\ \hat{\gamma}_{xy}^{[3]} \end{Bmatrix}, \quad (8)$$

$$\begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \hat{\gamma}_{xz}^{[0]} \\ \hat{\gamma}_{yz}^{[0]} \end{Bmatrix} + z^2 \begin{Bmatrix} \hat{\gamma}_{xz}^{[2]} \\ \hat{\gamma}_{yz}^{[2]} \end{Bmatrix},$$

where

$$\begin{Bmatrix} \hat{\varepsilon}_x^{[0]} \\ \hat{\varepsilon}_y^{[0]} \\ \hat{\gamma}_{xy}^{[0]} \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \bar{w}_{,x} w_{,x} + w_{,x}^2/2 \\ v_{,y} + \bar{w}_{,y} w_{,y} + w_{,y}^2/2 \\ u_{,y} + v_{,x} + w_{,x} w_{,y} + \bar{w}_{,y} w_{,x} + \bar{w}_{,x} w_{,y} \end{Bmatrix},$$

$$\begin{Bmatrix} \hat{\varepsilon}_x^{[1]} \\ \hat{\varepsilon}_y^{[1]} \\ \hat{\gamma}_{xy}^{[1]} \end{Bmatrix} = \begin{Bmatrix} \phi_{x,x} \\ \phi_{y,y} \\ \phi_{y,x} + \phi_{x,y} \end{Bmatrix}, \quad \lambda = \frac{4}{3h^2},$$

$$\begin{Bmatrix} \hat{\varepsilon}_x^{[3]} \\ \hat{\varepsilon}_y^{[3]} \\ \hat{\gamma}_{xy}^{[3]} \end{Bmatrix} = -\lambda \begin{Bmatrix} \phi_{x,x} + w_{,xx} \\ \phi_{y,y} + w_{,yy} \\ 2w_{,xy} + \phi_{y,x} + \phi_{x,y} \end{Bmatrix},$$

$$\begin{Bmatrix} \hat{\gamma}_{xz}^{[0]} \\ \hat{\gamma}_{yz}^{[0]} \end{Bmatrix} = \begin{Bmatrix} \phi_x + w_{,x} \\ \phi_y + w_{,y} \end{Bmatrix}, \quad \begin{Bmatrix} \hat{\gamma}_{xz}^{[2]} \\ \hat{\gamma}_{yz}^{[2]} \end{Bmatrix} = -3\lambda \begin{Bmatrix} \phi_x + w_{,x} \\ \phi_y + w_{,y} \end{Bmatrix}. \quad (9)$$

By extending the homogenization scheme for auxetic structures [23] to incorporate FG-CNTRC materials and thermal effects, the relations between the resultant force and moment components and the corresponding strain, rotation, and transverse deflection components of the plate are derived as follows.

$$N_x = \bar{L}_{11} \hat{\varepsilon}_x^{[0]} + \bar{L}_{12} \hat{\varepsilon}_y^{[0]} - \hat{\Phi}_{1x} \Delta T,$$

$$N_y = \bar{L}_{12} \hat{\varepsilon}_x^{[0]} + \bar{L}_{22} \hat{\varepsilon}_y^{[0]} - \hat{\Phi}_{1y} \Delta T,$$

$$N_{xy} = \bar{L}_{66} \hat{\gamma}_{xy}^{[0]},$$

$$M_x = \bar{F}_{11} \hat{\varepsilon}_x^{[1]} + \bar{F}_{12} \hat{\varepsilon}_y^{[1]} + \bar{A}_{11} \hat{\varepsilon}_x^{[3]} + \bar{A}_{12} \hat{\varepsilon}_y^{[3]},$$

$$M_y = \bar{F}_{12} \hat{\varepsilon}_x^{[1]} + \bar{F}_{22} \hat{\varepsilon}_y^{[1]} + \bar{A}_{12} \hat{\varepsilon}_x^{[3]} + \bar{A}_{22} \hat{\varepsilon}_y^{[3]}, \quad (10)$$

$$M_{xy} = \bar{F}_{66} \hat{\gamma}_{xy}^{[1]} + \bar{A}_{66} \hat{\gamma}_{xy}^{[3]},$$

$$T_x = \bar{A}_{11} \hat{\varepsilon}_x^{[1]} + \bar{A}_{12} \hat{\varepsilon}_y^{[1]} + \bar{D}_{11} \hat{\varepsilon}_x^{[3]} + \bar{D}_{12} \hat{\varepsilon}_y^{[3]},$$

$$T_y = \bar{A}_{12} \hat{\varepsilon}_x^{[1]} + \bar{A}_{22} \hat{\varepsilon}_y^{[1]} + \bar{D}_{12} \hat{\varepsilon}_x^{[3]} + \bar{D}_{22} \hat{\varepsilon}_y^{[3]},$$

$$T_{xy} = \bar{A}_{66} \hat{\gamma}_{xy}^{[1]} + \bar{D}_{66} \hat{\gamma}_{xy}^{[3]}.$$

The transverse shear forces and higher-order shear resultants, including Q_x , Q_y , S_x , and S_y , are expressed as follows.

$$Q_x = \bar{C}_{44} \hat{\gamma}_{xz}^{[0]} + \bar{B}_{44} \hat{\gamma}_{xz}^{[2]}, \quad Q_y = \bar{C}_{55} \hat{\gamma}_{yz}^{[0]} + \bar{B}_{55} \hat{\gamma}_{yz}^{[2]},$$

$$S_x = \bar{B}_{44} \hat{\gamma}_{xz}^{[0]} + \bar{E}_{44} \hat{\gamma}_{xz}^{[2]}, \quad S_y = \bar{B}_{55} \hat{\gamma}_{yz}^{[0]} + \bar{E}_{55} \hat{\gamma}_{yz}^{[2]}. \quad (11)$$

where

$$(\bar{L}_{ij}^{sh}, \bar{F}_{ij}^{sh}, \bar{A}_{ij}^{sh}, \bar{D}_{ij}^{sh}) = \int_{-h/2}^{h/2} \bar{Q}_{ij} (1, z^2, z^4, z^6) dz,$$

$$(i, j = 1, 2, 6),$$

$$(\bar{C}_{ij}^{sh}, \bar{B}_{ij}^{sh}, \bar{E}_{ij}^{sh}) = \int_{-h/2}^{h/2} \bar{Q}_{ij} (1, z^2, z^4) dz,$$

$$(i, j = 4, 5),$$

$$\hat{\Phi}_{1x} = \int_{-h/2}^{h/2} [\bar{Q}_{11}(z) \bar{\alpha}_{11}^{CNT}(z) + \bar{Q}_{12}(z) \bar{\alpha}_{22}^{CNT}(z)] dz,$$

$$\hat{\Phi}_{1y} = \int_{-h/2}^{h/2} [\bar{Q}_{12}(z) \bar{\alpha}_{11}^{CNT}(z) + \bar{Q}_{22}(z) \bar{\alpha}_{22}^{CNT}(z)] dz.$$

The compatibility equation derived from Eq. (9) is written in the following form

$$\hat{\varepsilon}_{x,yy}^{(0)} + \hat{\varepsilon}_{y,xx}^{(0)} - \hat{\gamma}_{xy,xy}^{(0)} = -w_{,yy} \bar{w}_{,xx} + 2w_{,xy} \bar{w}_{,xy} + w_{,xy}^2 - w_{,xx} w_{,yy} - \bar{w}_{,yy} w_{,xx} \quad (12)$$

The assumed stress function $\bar{\varsigma}(x,y)$ is required to satisfy the three conditions.

$$N_y = \bar{\varsigma}_{,xx}, \quad N_x = \bar{\varsigma}_{,yy}, \quad N_{xy} = -\bar{\varsigma}_{,xy} \quad (13)$$

By substituting Eqs. (10) and (13) into the strain compatibility equation (Eq. (12)), the following expression is obtained.

$$\bar{L}_{22} \bar{\varsigma}_{,xxxx} + (\bar{L}_{66} + 2\bar{L}_{12}) \bar{\varsigma}_{,xxyy} + \bar{L}_{11} \bar{\varsigma}_{,yyyy} + w_{,xx} \bar{w}_{,yy} - w_{,xy}^2 + w_{,xx} w_{,yy} + \bar{w}_{,xx} w_{,yy} - 2w_{,xy} \bar{w}_{,xy} = 0, \quad (14)$$

where

$$\bar{L}_{11} = \bar{L}_{22} / (\bar{L}_{11} \bar{L}_{22} - \bar{L}_{12}^2), \quad \bar{L}_{12} = -\bar{L}_{12} / (\bar{L}_{11} \bar{L}_{22} - \bar{L}_{12}^2), \\ \bar{L}_{22} = \bar{L}_{11} / (\bar{L}_{11} \bar{L}_{22} - \bar{L}_{12}^2), \quad \bar{L}_{66} = 1/\bar{L}_{66}.$$

4. Solution procedure, approximate stress function, and governing equations

In this study, two types of boundary conditions are considered for the plates. The first case corresponds to plates with all four edges simply supported and immovable (denoted as 4I), which can be expressed as follows.

$$\begin{aligned} \text{At } x=0,a: \quad N_x &= N_{0x}, \\ T_x &= 0, M_x = 0, u = w = 0, f_y = 0, \\ \text{At } y=0,b: \quad N_y &= N_{0y}, \\ T_y &= 0, M_y = 0, v = w = 0, f_x = 0. \end{aligned} \quad (15)$$

The second case corresponds to plates with all four edges simply supported, in which the edges at $y=0;b$ are allowed to move freely, while the edges at $x=0;a$ remain immovable (denoted as 2I2F), as defined below.

$$\begin{aligned} \text{at } x=0,a: \quad N_x &= N_{0x}, \\ T_x &= M_x = 0, \phi_x = 0, u = w = 0, \\ \text{at } y=0,b: \quad N_y &= N_{0y} = T_y = M_y = 0, \\ N_{xy} &= 0, \phi_x = 0, w = 0. \end{aligned} \quad (16)$$

To satisfy the specified boundary conditions, the displacement components, including the

transverse deflection, rotations, and initial geometric imperfection, are assumed in the following form [22]

$$\begin{aligned} w &= W \sin \bar{\alpha} x \sin \bar{\beta} y, \\ w^* &= \iota h \sin \bar{\alpha} x \sin \bar{\beta} y, \\ \phi_x &= \Phi_x \cos \bar{\alpha} x \sin \bar{\beta} y, \\ \phi_y &= \Phi_y \sin \bar{\alpha} x \cos \bar{\beta} y, \end{aligned} \quad (17)$$

where $\bar{\alpha} = m\pi/a$ and $\bar{\beta} = n\pi/b$, ι is the amplitude of the initial geometric imperfection, and m and n denote the half-wave numbers in the x - and y -directions, respectively.

By substituting Eq. (17) into Eq. (14), the stress function is obtained and expressed as follows

$$\bar{\varsigma} = \frac{1}{2} N_{0y} x^2 + \frac{1}{2} N_{0x} y^2 + \bar{\varsigma}_1 \cos 2\bar{\alpha} x + \bar{\varsigma}_2 \cos 2\bar{\beta} y. \quad (18)$$

where

$$\begin{aligned} \bar{\varsigma}_1 &= \bar{J}_{11} (W + 2\iota h) W, \quad \bar{\varsigma}_2 = \bar{J}_{21} (W + 2\iota h) W, \\ \bar{J}_{11} &= \frac{\bar{\beta}_1^2}{32\bar{\alpha}_1^2 \bar{L}_{22}}, \quad \bar{J}_{21} = \frac{\bar{\alpha}_1^2}{32\bar{\beta}_1^2 \bar{L}_{11}}. \end{aligned} \quad (19)$$

The constraints corresponding to immovable edges in the considered boundary conditions are enforced in an average sense as follows

$$\int_0^b \int_0^a u_{,x} dx dy = 0 \quad (20)$$

$$\bar{k}_{47} \Delta T + \bar{k}_{46} N_{0y} + \bar{k}_{45} N_{0x} + \bar{k}_{44} W (2\iota h + W) = 0,$$

$$\int_0^b \int_0^a v_{,y} dx dy = 0 \quad (21)$$

$$\bar{k}_{57} \Delta T + \bar{k}_{56} N_{0y} + \bar{k}_{55} N_{0x} + \bar{k}_{54} W (2\iota h + W) = 0,$$

where

$$\begin{aligned} \bar{k}_{44} &= -a\bar{\alpha}_1^2 b/8, \quad \bar{k}_{45} = ba\bar{L}_{11}, \\ \bar{k}_{46} &= ba\bar{L}_{12}, \quad \bar{k}_{47} = ab(\bar{L}_{11} \hat{\Phi}_{1x} + \bar{L}_{12} \hat{\Phi}_{1y}), \\ \bar{k}_{54} &= -a\bar{\beta}_1^2 b/8, \quad \bar{k}_{55} = ba\bar{L}_{22}, \\ \bar{k}_{56} &= ba\bar{L}_{22}, \quad \bar{k}_{57} = ab(\bar{L}_{12} \hat{\Phi}_{1x} + \bar{L}_{22} \hat{\Phi}_{1y}). \end{aligned}$$

The total potential energy can be expressed by summing the work performed by external loads and the strain energy, with the potential energy of the foundation, as follows.

$$\begin{aligned}\hat{\Xi}_{\text{Total}} &= \hat{\Xi}_{\text{in}} - \hat{\Xi}_{\text{ext}} \\ &= \frac{1}{2} \int_{-h/2}^{h/2} \int_0^b \int_0^a \left\{ \sigma_{xz} \gamma_{xz} + \sigma_{xy} \gamma_{xy} + \sigma_{yz} \gamma_{yz} + \right. \\ &\quad \left. \sigma_x [\varepsilon_x - \bar{\alpha}_{11}^{\text{CNT}}(z) \Delta T] + \sigma_y [\varepsilon_y - \bar{\alpha}_{22}^{\text{CNT}}(z) \Delta T] \right\} dx dy dz - \int_0^b \int_0^a \left\{ -\frac{1}{2} w [K_1 w - K_2 (w_{,xx} + w_{,yy})] \right\} dx dy.\end{aligned}\quad (22)$$

where K_1 (N/m^3) and K_2 (N/m) represent the Winkler and Pasternak foundation parameters, respectively.

By applying the Ritz energy method, the governing equations of the problem are obtained as follows

$$\frac{\partial \hat{\Xi}_{\text{Total}}}{\partial W} = 0, \quad \frac{\partial \hat{\Xi}_{\text{Total}}}{\partial \Phi_x} = 0, \quad \frac{\partial \hat{\Xi}_{\text{Total}}}{\partial \Phi_y} = 0, \quad (23)$$

leads to

$$\begin{aligned}\bar{k}_{11} W + \bar{k}_{12} \Phi_x + \bar{k}_{13} \Phi_y \\ + (W + ih) [\bar{k}_{17} W (W + ih) + \bar{k}_{19} N_{0x}] = 0,\end{aligned}\quad (24)$$

$$\bar{k}_{12} W + \bar{k}_{22} \Phi_x + \bar{k}_{23} \Phi_y = 0, \quad (25)$$

$$\bar{k}_{13} W + \bar{k}_{23} \Phi_x + \bar{k}_{33} \Phi_y = 0, \quad (26)$$

where

$$\bar{k}_{11} = \frac{ab}{4} \left\{ \lambda^2 \bar{D}_{11} \bar{\alpha}_1^4 + (2\bar{D}_{12} + 4\bar{D}_{66}) \lambda^2 \bar{\alpha}_1^2 \bar{\beta}_1^2 + \lambda^2 \bar{D}_{22} \bar{\beta}_1^4 + K_1 + K_2 (\bar{\alpha}_1^2 + \bar{\beta}_1^2) \right. \\ \left. + (9\bar{E}_{44} \lambda^2 - 6\bar{B}_{44} \lambda + \bar{C}_{44}) \bar{\alpha}_1^2 + (9\bar{E}_{55} \lambda^2 - 6\bar{B}_{55} \lambda + \bar{C}_{55}) \bar{\beta}_1^2 \right\},$$

$$\bar{k}_{12} = \frac{ab}{8} \left\{ \lambda (2\lambda \bar{D}_{11} - 2\bar{A}_{11}) \bar{\alpha}_1^3 + (18\lambda^2 \bar{E}_{44} - 12\bar{B}_{44} \lambda + 2\bar{C}_{44}) \bar{\alpha}_1 \right. \\ \left. + [(2\bar{D}_{12} + 4\bar{D}_{66}) \bar{\beta}_1^2 \lambda^2 - (2\bar{A}_{12} + 4\bar{A}_{66}) \bar{\beta}_1^2 \lambda] \bar{\alpha}_1 \right\},$$

$$\bar{k}_{13} = \frac{ab}{8} \left\{ \lambda (2\lambda \bar{D}_{22} - 2\bar{A}_{22}) \bar{\beta}_1^3 + (18\lambda^2 \bar{E}_{55} - 12\lambda \bar{B}_{55} + 2\bar{C}_{55}) \bar{\beta}_1 \right. \\ \left. + [(2\bar{D}_{12} + 4\bar{D}_{66}) \bar{\alpha}_1^2 \lambda^2 - (2\bar{A}_{12} + 4\bar{A}_{66}) \bar{\alpha}_1^2 \lambda] \bar{\beta}_1 \right\},$$

$$\bar{k}_{17} = 16ba (\bar{J}_{11}^2 \bar{\alpha}_1^4 \bar{L}_{22}^* + \bar{J}_{21}^2 \bar{\beta}_1^4 \bar{L}_{11}^*),$$

$$\bar{k}_{19} = \bar{\alpha}_1^2 ab/4,$$

$$\bar{k}_{22} = \frac{ab}{4} \left[(\lambda^2 \bar{D}_{11} - 2\bar{A}_{11} \lambda + \bar{F}_{11}) \bar{\alpha}_1^2 + (\lambda^2 \bar{D}_{66} - 2\bar{A}_{66} \lambda + \bar{F}_{66}) \bar{\beta}_1^2 + 9\bar{E}_{44} \lambda^2 - 6\bar{B}_{44} \lambda + \bar{C}_{44} \right],$$

$$\bar{k}_{23} = \frac{ab}{8} \bar{\alpha}_1 \bar{\beta}_1 \left[(2\bar{D}_{12} + 2\bar{D}_{66}) \lambda^2 + 2\bar{F}_{12} + 2\bar{F}_{66} - (4\bar{A}_{12} + 4\bar{A}_{66}) \lambda \right],$$

$$\bar{k}_{33} = \frac{ab}{4} \left[(\lambda^2 \bar{D}_{22} - 2\bar{A}_{22} \lambda + \bar{F}_{22}) \bar{\beta}_1^2 + (\lambda^2 \bar{D}_{66} - 2\bar{A}_{66} \lambda + \bar{F}_{66}) \bar{\alpha}_1^2 + 9\bar{E}_{55} \lambda^2 - 6\bar{B}_{55} \lambda + \bar{C}_{55} \right],$$

For the 2l2F boundary condition, the governing equation is derived by combining Eqs. (24)-(26) with Eq. (20) subject to $N_{0y} = 0$. In the case of the 4l boundary condition, the governing equation is obtained from Eqs. (24)-(26), (20),

and (21).

Therefore, the nonlinear thermal postbuckling response is represented in a unified form applicable to both boundary conditions, as follows.

$$\Delta T = - \frac{(\vartheta_3 + \vartheta_4) \bar{o}_{11} W + (\vartheta_3 \bar{o}_{35} + \vartheta_4 \bar{o}_{45}) W (W + ih) (2ih + W)}{(\vartheta_3 \bar{o}_{32} + \vartheta_4 \bar{o}_{42}) (ih + W)} \quad (27)$$

where ϑ_3 and ϑ_4 are parameters used to distinguish the boundary conditions, with $(\vartheta_3, \vartheta_4) = (1, 0)$ corresponding to the 2l2F case and $(0, 1)$

corresponding to the 4l case.

$$\bar{o}_{11} = \bar{k}_{12} \bar{T}_{14} + \bar{k}_{13} \bar{T}_{24} + \bar{k}_{11},$$

$$\bar{o}_{32} = \bar{T}_{44} \bar{k}_{19}, \quad \bar{o}_{35} = \bar{k}_{17} + \bar{T}_{41} \bar{k}_{19},$$

$$\bar{O}_{42} = -\frac{\bar{k}_{19}(\bar{T}_{34}\bar{T}_{43} + \bar{T}_{44})}{\bar{T}_{33}\bar{T}_{43} - 1},$$

$$\bar{O}_{45} = \bar{k}_{17} - \frac{\bar{k}_{19}(\bar{T}_{31}\bar{T}_{43} + \bar{T}_{41})}{\bar{T}_{33}\bar{T}_{43} - 1},$$

$$\bar{T}_{41} = -\frac{\bar{k}_{44}}{\bar{k}_{45}}, \quad \bar{T}_{43} = -\frac{\bar{k}_{46}}{\bar{k}_{45}}, \quad \bar{T}_{44} = -\frac{\bar{k}_{47}}{\bar{k}_{45}},$$

$$\bar{T}_{14} = -\frac{-\bar{k}_{33}\bar{k}_{12} + \bar{k}_{13}\bar{k}_{23}}{-\bar{k}_{22}\bar{k}_{33} + \bar{k}_{23}^2}, \quad \bar{T}_{24} = \frac{-\bar{k}_{12}\bar{k}_{23} + \bar{k}_{13}\bar{k}_{22}}{-\bar{k}_{22}\bar{k}_{33} + \bar{k}_{23}^2},$$

In the case of a perfect plate ($\iota = 0$), the critical thermal buckling load can be obtained from Eq. (27) by taking the limit $W \rightarrow 0$, obtained by

$$\Delta T_{cr} = -\frac{(\vartheta_3 + \vartheta_4)\bar{O}_{11}}{\vartheta_3\bar{O}_{32} + \vartheta_4\bar{O}_{42}} \quad (28)$$

Since the material properties are temperature-dependent, the coefficients in Eqs. (27) and (28) become functions of the temperature variation. Consequently, a fixed-point iterative scheme is adopted to determine the critical thermal buckling loads and to trace the corresponding

thermal load-deflection postbuckling equilibrium paths.

5. Results and discussion of nonlinear thermal behavior

This section provides numerical results for the thermal buckling and postbuckling responses of FG-CNTRC sandwich plates. The material system consists of a poly(methyl methacrylate) (PMMA) matrix reinforced with (10,10) single-walled CNT.

The temperature-dependent properties of the PMMA matrix are taken as $\bar{\nu}_M = 0.34$, $\bar{\alpha}_M = 45(1 + 0.0005T) \times 10^{-6}/K$, and $\bar{E}_M = (3.52 + 0.0034T)$, with $T = T_0 + \Delta T$ and $T_0 = 300K$ representing room temperature.

The temperature-dependent properties of SWCNTs are available at discrete temperature points in the literature [11] and are interpolated to construct continuous functions of temperature for use in the present model, presented by

$$\begin{aligned} \bar{\nu}_{12}^{CN} &= 0.175, \bar{E}_{11}^{CN} = -(2.2724 \times 10^{-9}T^3 - 4.22867 \times 10^{-6}T^2 + 0.00286T - 6.18387) \text{TPa}, \\ \bar{E}_{22}^{CN} &= -(-5.30057 \times 10^{-6}T^2 + 2.84868 \times 10^{-9}T^3 + 0.00358T - 7.75348) \text{TPa}, \\ \bar{G}_{12}^{CN} &= (1.80126 + 7.7845 \times 10^{-4}T + 4.93484 \times 10^{-10}T^3 - 1.1279 \times 10^{-6}T^2) \text{TPa}, \\ \bar{\alpha}_{11}^{CN} &= (-1.12148 + 0.02289T + 1.13253 \times 10^{-8}T^3 - 2.88155 \times 10^{-5}T^2) \times 10^{-6}/K, \\ \bar{\alpha}_{22}^{CN} &= (+3.13525 \times 10^{-7}T^2 - 3.56332 \times 10^{-12}T^3 - 9.95498 \times 10^{-4}T + 5.43874) \times 10^{-6}/K, \end{aligned} \quad (29)$$

Table 1. Comparison of critical thermal buckling temperatures $T_{cr} = T_0 + \Delta T_{cr}$ (K) for perfect FG-CNTRC plates under 4I boundary conditions with results reported in the literature ($V_{CNT}^* = 0.17$, $m = n = 1$, $T_0 = 300K$, $K_1 = 0 \text{ N/m}^3$, $K_2 = 0 \text{ N/m}$).

b/h		a/b = 1		a/b = 2	
		UD	FG-X	UD	FG-X
10	Shen and Zhang [11]	399.44 ^a (412.36 ^b)	419.09 (439.88)	363.67 (368.21)	381.51 (389.16)
	Tung and Trang [16]	399.01 (411.83)	419.50 (440.54)	363.82 (368.37)	381.60 (389.21)
	Present	391.90 (402.66)	409.25 (427.21)	365.45 (370.83)	382.25 (391.85)
20	Shen and Zhang [11]	343.00 (344.54)	359.52 (362.79)	319.01 (319.34)	326.16 (326.77)
	Tung and Trang [16]	342.96 (344.52)	359.75 (363.00)	319.01 (319.36)	325.92 (326.60)
	Present	339.38 (340.65)	353.73 (357.00)	319.64 (320.05)	326.44 (327.46)

^aT-D: temperature-dependent; ^bT-ID: temperature-independent

Table 1 presents a comparison of the critical thermal buckling temperatures of perfect FG-CNTRC plates obtained in the present study with those reported in the literature. It can be observed that the present results are in good agreement with

the reference solutions, with only minor discrepancies across different plate geometries and CNT distribution patterns. This consistency confirms the accuracy and reliability of the proposed analytical formulation.

Table 2. Effects of CNT volume fraction, distribution patterns, and boundary conditions on the critical thermal buckling temperature increase ΔT_{cr} (K) of FG-CNTRC sandwich plates with auxetic core

($h=0.02m$, $h_{AU}=80h$, $a=b=20h$, $(m,n)=(1,1)$, $\vartheta_1=2$, $\vartheta_3=0.1$, $\delta=30^\circ$, $K_1=10\text{ MN/m}^3$, $K_2=0.1\text{ MN/m}$)

	V_{CNT}^*	X-AU(X)-X	UD-AU(UD)-UD	O-AU(O)-O
2I2F	0.12	126.41 (134.16)	121.72 ^a (128.66) ^b	126.01 (133.69)
	0.17	129.74 (138.54)	122.59 (130.07)	129.31 (138.04)
	0.28	146.29 (158.08)	131.13 (139.75)	145.76 (157.46)
4I	0.12	129.91 (138.54)	124.98 (132.69)	129.50 (138.07)
	0.17	133.71 (143.60)	126.17 (134.52)	133.27 (143.07)
	0.28	149.98 (163.02)	134.16 (143.62)	149.44 (162.38)

^aT-D: temperature-dependent; ^bT-ID: temperature-independent

Table 3. Comparison of the effects of FG-CNTRC and PMMA auxetic cores on the critical thermal buckling temperature increase ΔT_{cr} (K) of sandwich plates under different CNT distribution patterns and

boundary conditions ($h=0.02m$, $h_{AU}=80h$, $a=b=20h$, $V_{CNT}^*=0.17$, $(m,n)=(1,1)$, $\vartheta_1=2$, $\vartheta_3=0.1$, $\delta=30^\circ$, $K_1=10\text{ MN/m}^3$, $K_2=0.1\text{ MN/m}$)

		X-AU(X)-X	UD-AU(UD)-UD	O-AU(O)-O
FG-CNTRC	2I2F	129.74 (138.54)	122.59 (130.07)	129.31 (138.04)
Auxetic core	4I	133.71 (143.60)	126.17 (134.52)	133.27 (143.07)
PMMA	2I2F	64.73 (69.70)	60.63 (64.76)	63.91 (68.74)
Auxetic core	4I	64.07 (68.86)	60.04 (64.02)	63.26 (67.91)

^aT-D: temperature-dependent; ^bT-ID: temperature-independent

Table 2 shows that the critical thermal buckling temperature increase ΔT_{cr} is markedly affected by the CNT volume fraction, the CNT distribution pattern, and the boundary condition. For all distribution schemes and both boundary conditions, ΔT_{cr} increases monotonically as the CNT volume fraction rises from 0.12 to 0.28. This trend indicates that a higher CNT content substantially enhances the effective stiffness of both the FG-CNTRC face sheets and the auxetic core, thereby improving the resistance of the sandwich plate to thermally induced instability. Among the three considered configurations, the X-

AU(X)-X pattern consistently gives the highest critical thermal buckling temperature, whereas the UD-AU(UD)-UD case yields the lowest values, and the O-AU(O)-O pattern lies very close to the X-type configuration. For example, under the 4I boundary condition at $V_{CNT}^*=0.28$, the critical temperature increase reaches 149.98 K for X-AU(X)-X, compared with 149.44 K for O-AU(O)-O and 134.16 K for UD-AU(UD)-UD. The influence of the boundary condition is likewise significant. For each material configuration, the 4I plates exhibit higher critical thermal buckling temperatures than the 2I2F plates. This result is mechanically reasonable

because the more constrained 4I boundary condition provides stronger in-plane restraint and suppresses thermally induced deflection more effectively, resulting in greater thermal stability. Another important observation is that all temperature-dependent results are lower than their temperature-independent counterparts. This difference becomes more pronounced at larger CNT volume fractions and for the X- and O-type distributions. Such a trend reflects the thermal degradation of material stiffness as the temperature rises. Therefore, neglecting temperature dependence would overestimate the critical thermal buckling capacity of the structure, especially for highly reinforced configurations.

Table 3 highlights the significant influence of the auxetic core material on the critical thermal buckling temperature increase ΔT_{cr} of the sandwich plates. A very clear trend can be observed: for all CNT distribution patterns and for both boundary conditions, the plates with an FG-CNTRC auxetic core exhibit substantially higher critical thermal buckling temperatures than those with a PMMA auxetic core. For instance, under the 4I boundary condition, the X-AU(X)-X configuration gives $\Delta T_{cr} = 133.71$ K for the FG-CNTRC core, whereas the corresponding value is only 64.07 K for the PMMA core. This nearly twofold increase demonstrates the remarkable contribution of the CNT-reinforced auxetic core to the enhancement of thermal stability. This behavior can be explained by the much higher stiffness of the FG-CNTRC auxetic core compared with the PMMA core. As a result, the sandwich plate becomes more resistant to thermally induced deformation, leading to a considerable increase in the critical thermal buckling temperature.

Fig. 2 illustrates the effects of the auxetic core thickness and the geometrical parameter ϑ_1 on the thermal postbuckling behavior of the FG-CNTRC sandwich plates. A common feature observed in all subfigures is that the thermal load-

carrying capacity is strongly dependent on the auxetic-core configuration, confirming the essential role of the core in governing the nonlinear thermal stability of the sandwich system. As shown in Figs. 2(a) and 2(b), increasing the auxetic core thickness leads to a clear upward shift of the thermal postbuckling curves for both perfect and imperfect plates. This indicates that a thicker auxetic core enhances the thermal buckling resistance and improves the postbuckling strength of the structure. Figs. 2(c) and 2(d) show that the geometrical parameter ϑ_1 also has a pronounced influence on the thermal postbuckling response. As ϑ_1 increases, the postbuckling curves generally move downward, indicating a reduction in the critical thermal buckling temperature and thermal postbuckling resistance. Another important feature observed in Fig. 2 is the fundamental difference between perfect and imperfect plates in their thermal postbuckling responses. For perfect plates, a classical bifurcation behavior is clearly exhibited. The structure initially remains in a pre-buckling membrane state with negligible transverse deflection, and once the critical temperature is reached, a bifurcation point occurs, leading to a sudden transition to a postbuckling equilibrium path. In contrast, imperfect plates do not exhibit a bifurcation-type response. Due to the presence of initial geometric imperfections, the plates undergo a gradual and continuous increase in transverse deflection as the temperature rises, without a distinct membrane state or a sharp bifurcation point.

Fig. 3 presents the effects of boundary conditions and edge length on the thermal postbuckling behavior of FG-CNTRC sandwich plates with an auxetic core. It is evident that both parameters strongly influence not only the critical thermal buckling temperature but also the subsequent nonlinear equilibrium path. As shown in Figs. 3(a) and 3(b), the boundary condition has a pronounced effect on the thermal stability

response. The plates under the 4I boundary condition exhibit higher thermal buckling resistance and stiffer postbuckling paths than those under the 2I2F condition. For perfect plates, this difference is manifested through a higher bifurcation point for the 4I case, whereas for imperfect plates the entire nonlinear response curve is shifted upward when stronger edge constraints are imposed. Figs. 3(c) and 3(d) show that the edge length also plays an

important role in the thermal postbuckling response. As the plate becomes larger in the in-plane direction, the thermal buckling resistance decreases and the postbuckling curves move downward. This trend can be explained by the reduction in the overall structural stiffness associated with a larger span, which makes the plate more susceptible to thermally induced out-of-plane deformation.

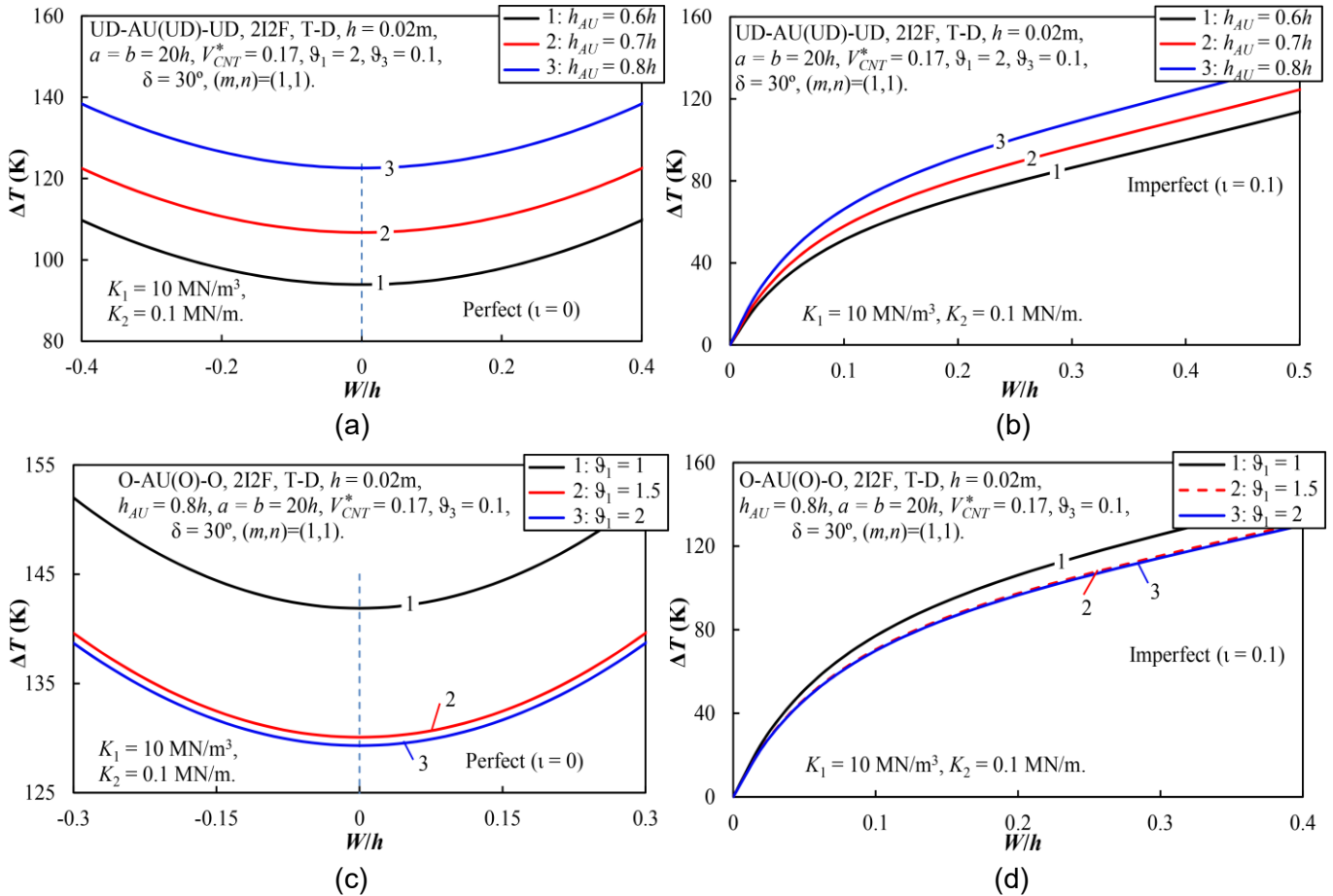


Fig. 2. Influences of auxetic core thickness and geometrical parameter ϑ_1 of auxetic core on the thermal postbuckling curves of plates. (a) Effects of auxetic core thickness (perfect plates); (b) Effects of auxetic core thickness (imperfect plates); (c) Effects of ϑ_1 (perfect plates); (d) Effects of ϑ_1 (imperfect plates)

Fig. 4 illustrates the effects of temperature-dependent material properties and foundation parameters on the thermal postbuckling behavior of FG-CNTRC sandwich plates with an auxetic core. As shown in Figs. 4(a) and 4(b), the curves obtained by considering temperature-independent material properties lie consistently above those corresponding to temperature-dependent

properties for both perfect and imperfect plates. This indicates that neglecting the temperature dependence of the constituent properties leads to an overestimation of the thermal buckling resistance. The physical reason is that, when the temperature rises, the effective elastic moduli of the matrix and CNT-reinforced composite decrease, which reduces the overall stiffness of the

sandwich plate and makes thermal instability occur at a lower temperature level. In addition, for a given material assumption, increasing the CNT volume fraction from 0.12 to 0.28 shifts the postbuckling curves upward, confirming that a higher CNT content enhances the stiffness of both the face sheets and the auxetic core and therefore improves the thermal stability of the structure. Figs. 4(c) and 4(d) show the influence of the elastic

foundation parameters. In both perfect and imperfect cases, increasing the foundation stiffness causes the thermal postbuckling curves to move upward. The plate resting on the stronger Pasternak foundation therefore requires a higher temperature rise to reach the same transverse deflection. This behavior is expected because the elastic foundation provides additional restraint against out-of-plane deformation.

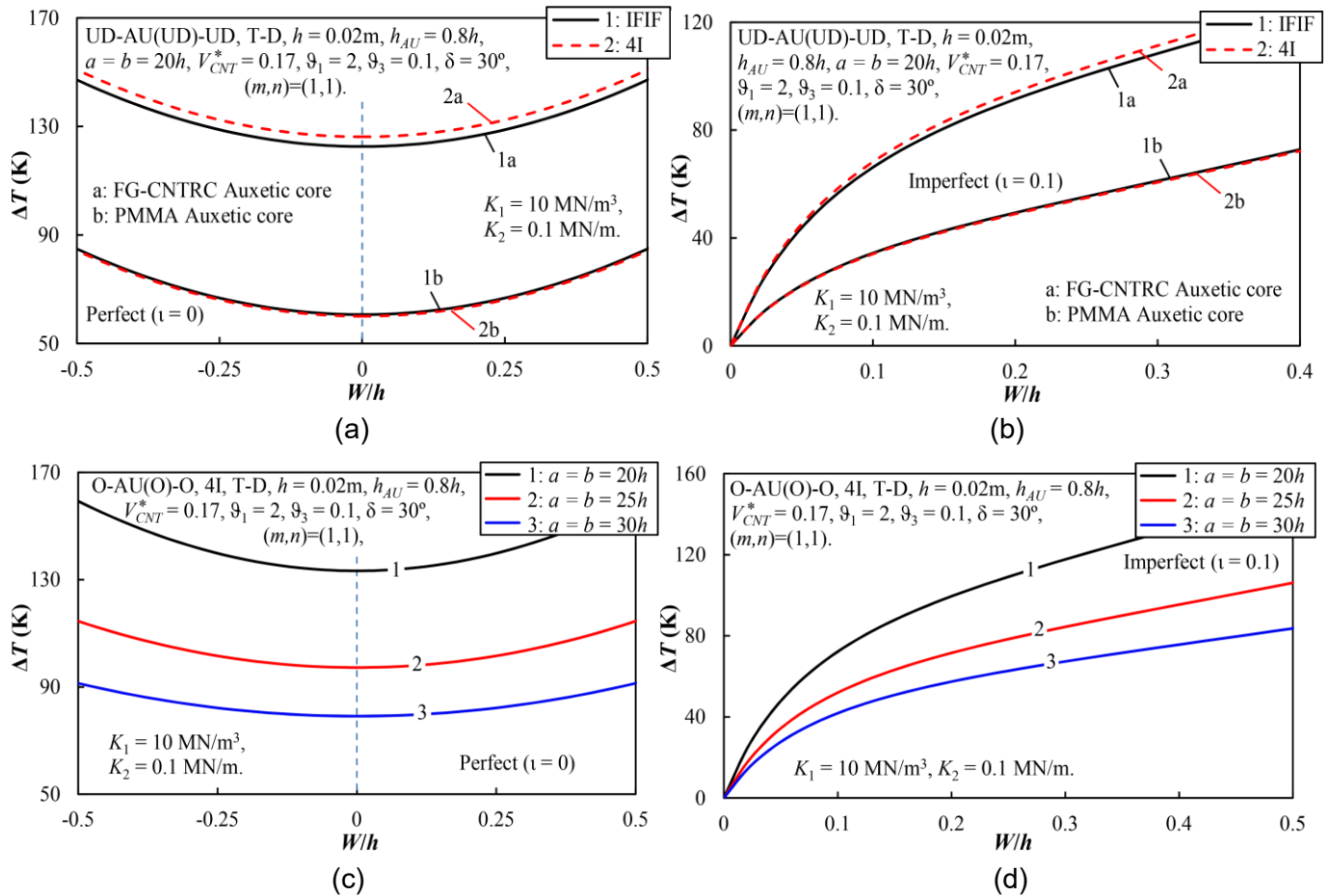


Fig. 3. Influences of boundary condition and edge length on the thermal postbuckling curves of plates. (a) Effects of boundary condition (perfect plates); (b) Effects of boundary condition (imperfect plates); (c) Effects of edge length (perfect plates); (d) Effects of edge length (imperfect plates)

6. Conclusions

An analytical investigation has been presented for the nonlinear thermal buckling and postbuckling behavior of FG-CNTRC sandwich plates with an auxetic FG-CNTRC core resting on a two-parameter Pasternak elastic foundation. The formulation was established on the basis of higher-order shear deformation theory, von Kármán geometric nonlinearity, an energy-based

approach, and a homogenization scheme for the auxetic core in which the CNTs were assumed to be aligned along the cell walls. Temperature-dependent material properties were incorporated into the constitutive model, and explicit expressions for the critical thermal buckling temperature and nonlinear thermal postbuckling response were obtained. The numerical results confirmed the accuracy of the present formulation

through comparison with available results in the literature. The parametric study further led to the following main conclusions:

1. The critical thermal buckling temperature increases significantly with increasing CNT volume fraction. The CNT distribution pattern has a pronounced influence on thermal stability. Among

the considered cases, the X-AU(X)-X configuration generally provides the highest critical thermal buckling temperature, whereas the UD-AU(UD)-UD configuration gives the lowest values. The use of an FG-CNTRC auxetic core leads to a substantial improvement in thermal stability compared with a conventional PMMA auxetic core.

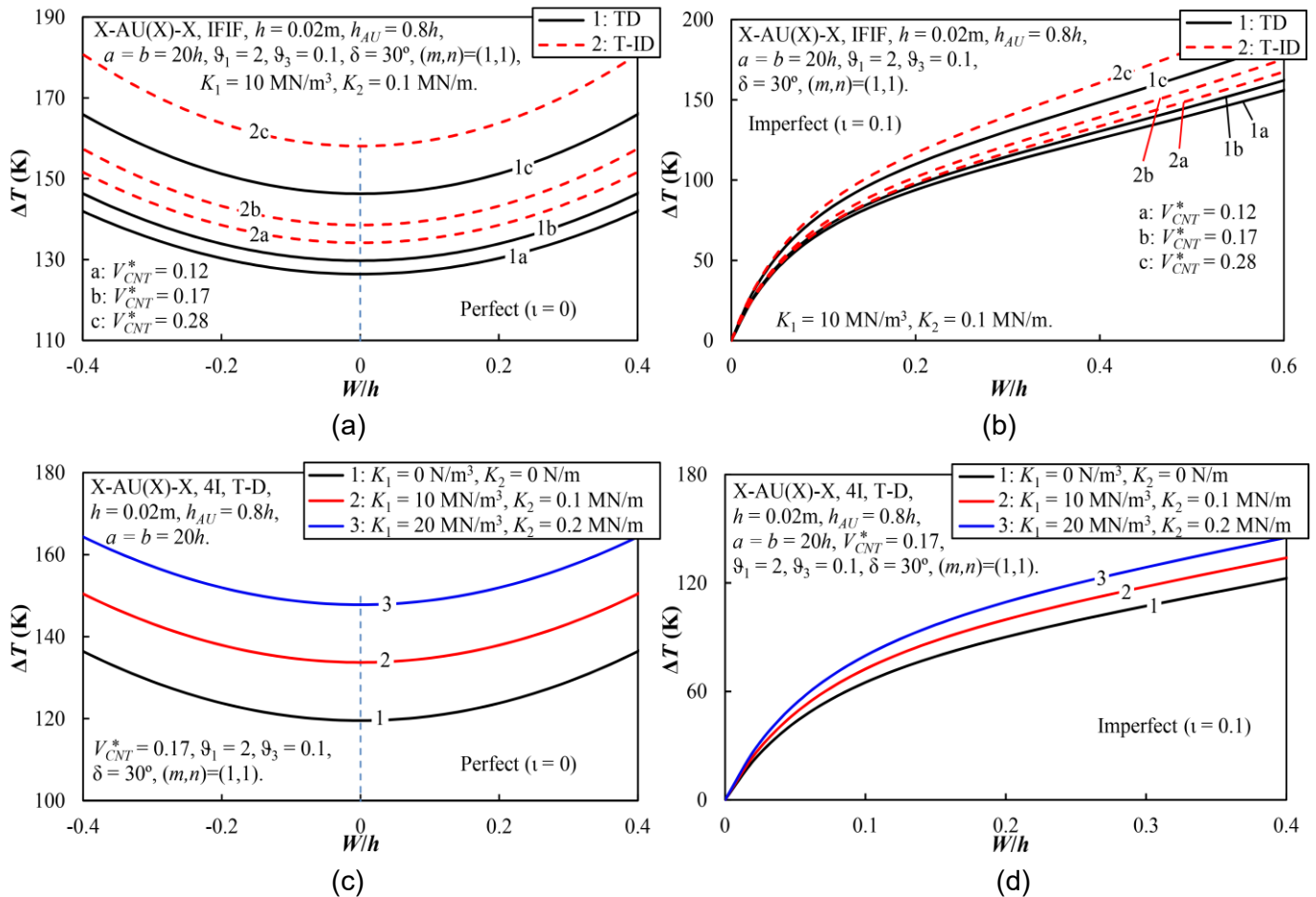


Fig. 4. Influences of dependent/independent properties of materials and foundation parameters on the thermal postbuckling curves of plates. (a) Effects of dependent/independent properties of materials (perfect plates); (b) Effects of dependent/independent properties of materials (imperfect plates); (c) Effects of foundation parameters (perfect plates); (d) Effects of foundation parameters (imperfect plates)

2. Increasing the auxetic core thickness enhances the thermal postbuckling resistance of the plate, whereas increasing the geometrical parameter ϑ_1 reduces it. These parameters directly affect the equivalent stiffness of the auxetic core and therefore provide an effective means for tailoring the nonlinear thermal stability of the sandwich system.

3. Boundary conditions and plate dimensions

strongly affect the postbuckling response. Temperature-dependent solutions are consistently lower than the corresponding temperature-independent predictions, showing that neglecting thermal degradation of material properties may lead to unconservative estimates of buckling resistance.

Declaration of Generative AI

During the preparation of this work, the

authors did not use Generative AI and AI-assisted technology tools in order to generate the content of the manuscript.

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