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An analytical approach for radial-pressure buckling responses of helically corrugated FG-GRMMC cylindrical shells with temperature effects

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Abstract: This study presents an analytical formulation for the nonlinear buckling and postbuckling behavior of helically corrugated cylindrical shells made of functionally graded graphene-reinforced metal matrix composites (FG-GRMMC) under external pressure in a thermal environment. The governing equations are established based on the Donnell shell theory incorporating von Kármán geometric nonlinearity. An equivalent-shell homogenization approach is developed to represent the helical corrugation, in which temperature-dependent material properties and effective thermal force resultants are consistently incorporated. The resulting nonlinear equilibrium system is solved using the Ritz energy method with a three-term assumed deflection function. Closed-form expressions for the critical buckling loads and postbuckling response are obtained. Numerical results are presented to examine the influences of graphene distribution patterns, corrugation geometry, and temperature variation on the stability characteristics of the shells. The results demonstrate that helical corrugation significantly enhances the global stability, while increasing temperature reduces the load-carrying capacity.

Keywords: Functionally graded graphene reinforced metal matrix composite (FG-GRMMC), Helically corrugated cylindrical shells, Postbuckling behavior, External pressure, Analytical approach.

1. Introduction

Functionally graded materials (FGMs) have attracted considerable attention in advanced engineering applications due to their superior mechanical performance and the ability to tailor material properties continuously through the thickness. Such characteristics make FGMs particularly suitable for high-performance structures in aerospace, nuclear engineering, and

civil infrastructure. Among these structures, cylindrical shells fabricated from FGMs have been extensively studied, especially with respect to their stability and vibration behaviors under various loading and environmental conditions.

A significant body of research has been devoted to the buckling and postbuckling responses of FGM cylindrical shells. The nonlinear postbuckling behavior of FGM cylindrical shells

under axial compression and external pressure in thermal environments was investigated by Shen [1, 2], employing the Donnell shell theory combined with von Kármán nonlinearities and accounting for prebuckling deformation and initial imperfections. Further developments were reported by Shen and Xiang [3], who analyzed the nonlinear buckling of piezolaminated cylindrical shells considering temperature-dependent material properties using the higher-order shear deformation theory (HSDT). Using energy-based approaches, both nonlinear and linear buckling responses of perfect and imperfect FGM cylindrical shells under axial compression were investigated by Huang and Han [4, 5]. In addition, Sofiyev and co-workers [6, 7] provided comprehensive studies on the stability and vibration of FGM cylindrical shells under static and dynamic loading conditions. The effects of elastic foundations were also addressed in the nonlinear buckling behavior of FGM cylindrical shells resting on Pasternak-type foundations under combined loading by Bagherizadeh et al. [8] using the HSDT.

In recent years, a class of advanced nanocomposites, namely functionally graded carbon nanotube-reinforced composites (FG-CNTRCs), has emerged as a promising alternative to conventional FGMs [9–11]. Owing to the exceptional thermo-mechanical properties of carbon nanotubes (CNTs), such as high stiffness and strength, their incorporation into a matrix material can significantly enhance the overall structural performance. Moreover, the continuous and graded distribution of CNTs through the thickness enables an efficient utilization of their load-carrying capability, leading to improved mechanical efficiency compared to uniformly reinforced composites.

Similar to conventional FGMs, extensive research efforts have been devoted to the analysis of FG-CNTRC cylindrical shells, including their buckling, postbuckling, free vibration, and static responses under various loading conditions [9–13]. It has been shown that the incorporation of CNTs

introduces pronounced anisotropic characteristics, which significantly influence the structural behavior of cylindrical shells, particularly in terms of stability and deformation responses. The combined effects of material anisotropy and structural complexity give rise to distinct mechanical features that differ from those of traditional composite shells. To effectively capture these characteristics, advanced modeling approaches have been developed. For instance, an anisotropic smeared stiffener technique was improved to represent FG-CNTRC stiffeners, enabling accurate prediction of the buckling behavior of cylindrical shells with tailored CNT distributions in both the shell and stiffening elements [14, 15]. Furthermore, FG-CNTRC materials have been utilized as face sheets in sandwich structures, often combined with auxetic cores to enhance load-carrying capacity and structural efficiency [16, 17]. The viscoelastic response of FG-CNTRC cylindrical shells under seismic excitations has also been investigated, taking into account hygrothermal effects [18, 19]. In addition, laminated configurations of FG-CNTRC shells have been explored, including the influence of a negative Poisson's ratio on their mechanical performance [20].

More recently, the emergence of graphene as a nanoscale reinforcement material with exceptional thermo-mechanical properties has opened new opportunities for the development of high-performance composite structures. Due to its remarkable stiffness, strength, and thermal stability, graphene has been widely recognized as an efficient reinforcing phase for metallic and polymeric matrices. By integrating the concept of functional grading, functionally graded graphene-reinforced composites (FG-GRCs) have been developed, offering enhanced and tunable structural performance [21]. Extensive studies have been conducted to explore the mechanical behavior of FG-GRC cylindrical shells under various loading conditions. The perturbation method has been employed to investigate their nonlinear buckling and postbuckling responses

under axial compression, external pressure, and torsional loading, taking into account higher-order shear deformation effects [21–23]. The concept of adjacent equilibrium has also been utilized to derive explicit expressions for critical buckling loads of FG-GRC conical shells with first-order shear deformation and associated boundary conditions [24]. In addition, FG-GRC toroidal shell segments have been analyzed using the Donnell shell theory in conjunction with the Galerkin method, including the influence of shell-foundation interaction [25]. Furthermore, the stability behavior of stiffened FG-GRC cylindrical shells has been examined under different loading scenarios, such as torsion, external pressure, and axial compression, by employing improved smeared stiffener techniques [26–28]. Similar analytical frameworks have also been extended to investigate the nonlinear torsional buckling of FG-GRC toroidal shells with auxetic cores through homogenization-based approaches [29]. In addition to polymer-based graphene composites, graphene-reinforced metal matrix composites (GRMMCs) have attracted increasing attention due to their superior mechanical strength and thermal resistance. By combining graphene reinforcement with functionally graded metal matrices, FG-GRMMCs offer enhanced stiffness and stability, particularly in high-temperature environments [30]. However, studies on FG-GRMMC cylindrical shells, especially those involving complex geometries and thermal effects, remain limited.

Corrugated panels represent a class of engineered structures characterized by periodically varying geometries whose wavelength is significantly smaller than the overall structural dimensions. This geometric configuration induces pronounced directional stiffness, resulting in highly anisotropic mechanical behavior. To facilitate the analysis of such structures, homogenization techniques are commonly employed to replace the corrugated geometry with an equivalent anisotropic continuum. A number of studies have been devoted to the development and application of such

equivalent models. For instance, a homogenization approach for corrugated panels made of orthotropic materials, neglecting transverse shear effects, was proposed in [31]. Based on this framework, the vibration behavior of corrugated cylindrical shells composed of symmetric FGMs and filled with fluid was subsequently investigated [32]. Other works have addressed the nonlinear dynamic responses of corrugated plates and thin films under large-amplitude oscillations [33, 34]. In addition, homogenization-based models have been extended to more complex configurations, such as toroidal shell segments with corrugated cores and FG-GRC face sheets subjected to torsional and axial loads [35, 36], as well as multilayer FG-CNTRC cylindrical panels with corrugated cores in nonlinear buckling analyses [37]. More recently, the homogenization technique has been combined with coordinate transformation methods to analyze helically corrugated cylindrical shells. In particular, the nonlinear buckling of FG-CNTRC cylindrical shells with helically corrugated configurations under external pressure was investigated in [38]. However, these studies primarily focus on mechanical loading conditions, while the influence of thermal environments on the stability behavior of helically corrugated nanocomposite shells has not yet been sufficiently addressed.

Despite the considerable progress in the analysis of functionally graded and graphene-reinforced composite cylindrical shells, several important issues remain unresolved. Existing studies on corrugated and helically corrugated shell structures have primarily focused on polymer-based composites and mechanical loading conditions, while the combined effects of temperature, material gradation, and complex corrugated geometries have not been fully addressed. In particular, the nonlinear stability behavior of helically corrugated FG-GRMMC cylindrical shells in thermal environments has received little attention in the literature. Furthermore, although homogenization techniques

have been widely employed for corrugated structures, their extension to thermo-mechanical problems involving FG-GRMMC materials remains limited. Most existing analytical studies are based on Galerkin-type approaches, whereas the application of the Ritz energy method to such complex shell systems has not been sufficiently explored. In addition, helically corrugated cylindrical shells offer notable advantages over conventional ring-corrugated configurations in terms of both structural performance and manufacturability. From a practical perspective, such structures can be efficiently fabricated using continuous helical winding techniques, making them attractive for engineering applications such as aerospace pressure vessels, subsea pipelines, and high-temperature structural components.

Motivated by these observations, the present study develops an analytical formulation for the nonlinear buckling responses of helically corrugated FG-GRMMC cylindrical shells subjected to external pressure in a thermal environment. The model incorporates an equivalent-shell homogenization scheme with temperature-dependent material properties, combined with a coordinate transformation technique to account for the helical geometry. The governing equations are solved using the Ritz method, enabling the evaluation of both critical buckling loads and postbuckling responses.

2. Thermo-Mechanical Equivalent Modeling of Helically Corrugated FG-GRMMC Cylindrical Shells

In this paper, the helically corrugated FG-GRMMC cylindrical shells with radius R , thickness h , and length L , in which graphene reinforcements are aligned along the corrugation direction. According to the manufacturing process, the shell is formed by winding a corrugated strip into a helical configuration. The corrugation profile is assumed to be either circular or trapezoidal, as illustrated in Fig. 1. A local coordinate system (n,s,z) is introduced, where the s -direction follows

the corrugated helical path, and the graphene reinforcements are oriented along this direction.

Based on geometric considerations, the number of corrugation waves per strip, denoted by N , can be expressed as a function of the corrugation angle as follows

$$\theta = \arcsin\left(\frac{Nc_c}{2\pi R}\right) \quad (1)$$

where c_c represents the half-wavelength of the corrugation profile.

The geometric parameters of the corrugation are assumed to be small compared to the overall dimensions of the cylindrical shell. Under this assumption, the corrugated structure can be approximated by an equivalent homogeneous model for analyzing the global buckling responses of the shell.

To describe the helically corrugated cylindrical shell, a global coordinate system (x,y,z) is employed. The mid-surface at one end of the shell is selected as the origin. The x -axis corresponds to the axial direction, while the circumferential direction is represented by the y -axis with $y = R\theta$; the z -axis is defined as the inward normal to the mid-surface.

Graphene is embedded in the copper (Cu) matrix with volume fractions ranging from 0.05 to 0.13, and is discretized into ten layers along the shell thickness. Several functionally graded (FG) distribution patterns are considered. The FG-V distribution is defined by the layer-wise volume fractions $[(0.13)_2/(0.11)_2/(0.09)_2/(0.07)_2/(0.05)_2]$, while the FG-A distribution follows the reverse sequence $[(0.05)_2/(0.07)_2/(0.09)_2/(0.11)_2/(0.13)_2]$.

In addition, symmetric distributions are introduced, including the FG-X pattern $[(0.13)/(0.11)/(0.09)/(0.07)/(0.05)]_s$ and the FG-O pattern $[(0.05)/(0.07)/(0.09)/(0.11)/(0.13)]_s$. For comparison, a uniformly distributed (UD) case with a constant volume fraction of $[0.09]_{10}$ is also considered.

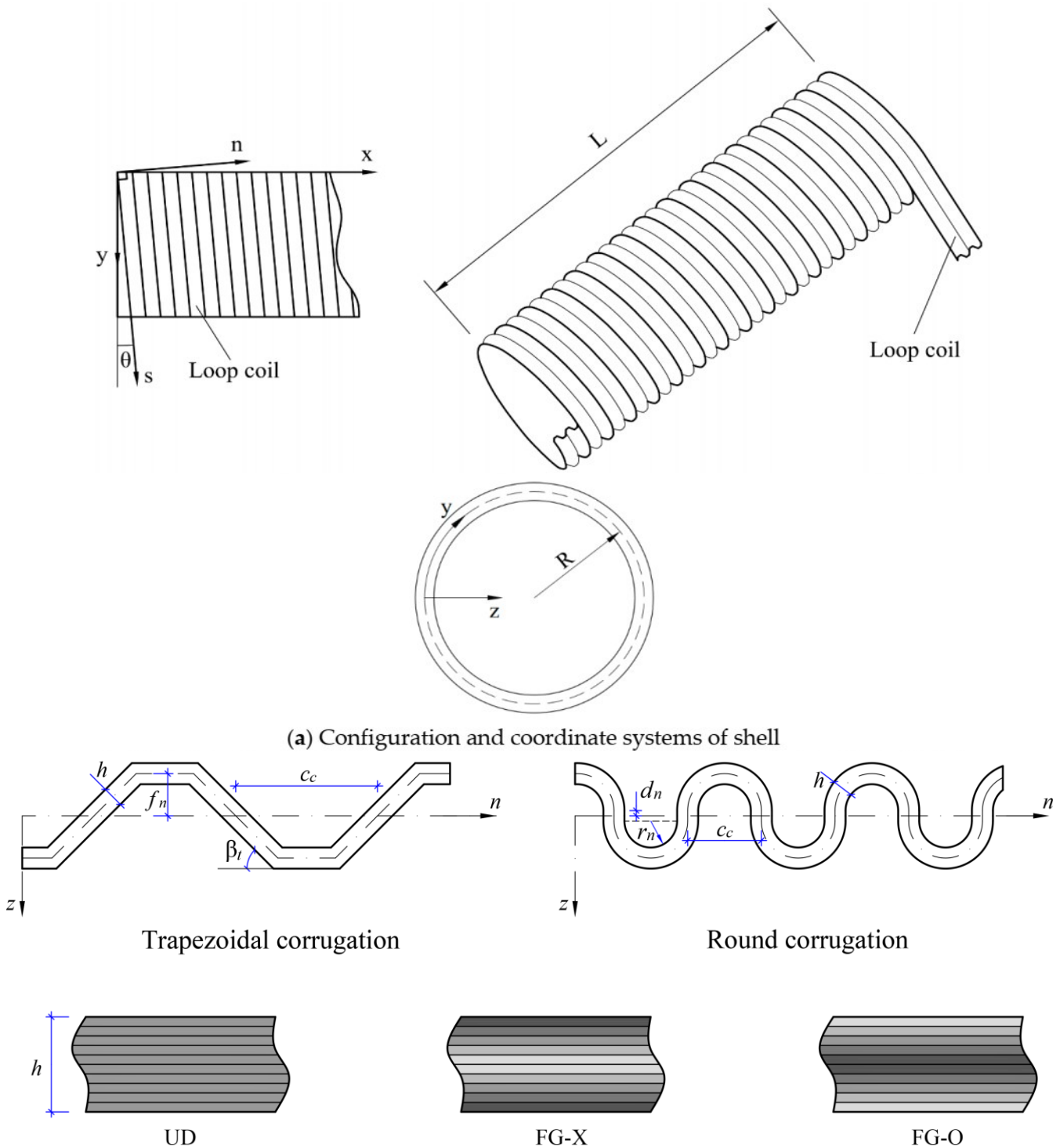


Fig. 1. Geometry and coordinate systems of helically corrugated FG-GRMMC cylindrical shells

In the present formulation, the stiffening configuration is defined consistently with the material gradation of the shell surface. The graphene reinforcements are assumed to be uniformly oriented along the local s -direction, which is represented by a stacking sequence of $(0)_{10}$. The thermo-elastic material parameters of the GRMMC are obtained from Fan et al. [30].

The stress-strain relations for the non-corrugated FG-GRMMC shell, formulated in the local coordinate system, can be expressed as follows

$$\begin{bmatrix} \sigma_n \\ \sigma_s \\ \sigma_{ns} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_n \\ \varepsilon_s \\ \gamma_{ns} \end{bmatrix} - \begin{bmatrix} \alpha_{11}^n \\ \alpha_{22}^s \\ 0 \end{bmatrix} \Delta T, \quad (2)$$

where Q_{ij} are the reduced stiffnesses, presented as

$$Q_{11} = \frac{E_{11}}{1 - \nu_{21}\nu_{12}}, \quad Q_{22} = \frac{E_{22}}{1 - \nu_{21}\nu_{12}},$$

$$Q_{12} = \frac{\nu_{21}E_{11}}{1 - \nu_{21}\nu_{12}}, \quad Q_{66} = G_{12}.$$

Based on the homogenization approach for corrugated panels proposed by Xia et al. [31], and adopting the kinematic assumptions of Kirchhoff shell theory, the internal force resultants of the FG-GRMMC shell in the local coordinate system can be derived as follows

$$\begin{bmatrix} N_n \\ N_s \\ N_{ns} \\ M_n \\ M_s \\ M_{ns} \end{bmatrix} = \begin{bmatrix} \hat{A}_{11}^* & \hat{A}_{12}^* & 0 & 0 & 0 & 0 \\ \hat{A}_{12}^* & \hat{A}_{22}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & \hat{A}_{66}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & \hat{D}_{11}^* & \hat{D}_{12}^* & 0 \\ 0 & 0 & 0 & \hat{D}_{12}^* & \hat{D}_{22}^* & 0 \\ 0 & 0 & 0 & 0 & 0 & \hat{D}_{66}^* \end{bmatrix} \times$$

$$\begin{bmatrix} \bar{\epsilon}_n^0 \\ \bar{\epsilon}_s^0 \\ \bar{\gamma}_{ns}^0 \\ \chi_n \\ \chi_s \\ 2\chi_{ns} \end{bmatrix} - \begin{bmatrix} \hat{\Phi}_{1n} \\ \hat{\Phi}_{1s} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T, \quad (3)$$

where the stiffness coefficients of the ring-corrugated shell are defined as follows

$$\hat{A}_{11}^* = \frac{2c_c}{\bar{I}_1 + \bar{I}_2}, \quad \hat{A}_{12}^* = \frac{\hat{A}_{12}}{\hat{A}_{11}} \hat{A}_{11}^*,$$

$$\hat{A}_{22}^* = \frac{\hat{A}_{12}^* \hat{A}_{12}}{\hat{A}_{11}} + \frac{I_c}{c_c} \frac{\hat{A}_{11} \hat{A}_{22} - \hat{A}_{12}^2}{\hat{A}_{11}}, \quad \hat{A}_{66}^* = \frac{c_c}{I_c} \hat{A}_{11},$$

$$\hat{D}_{11}^* = \frac{c_c}{I_c} \hat{D}_{11}, \quad \hat{D}_{12}^* = \frac{\hat{D}_{12}}{\hat{D}_{11}} \hat{D}_{11}^*,$$

$$\hat{D}_{22}^* = \frac{1}{2c_c} (\bar{I}_2 \hat{A}_{22} + \bar{I}_1 \hat{D}_{22}), \quad \hat{D}_{66}^* = \frac{I_c}{c_c} \hat{D}_{66},$$

$$\hat{\Phi}_{1n} = \alpha_{11}^n \hat{A}_{11}^* + \alpha_{22}^s \hat{A}_{12}^*, \quad \hat{\Phi}_{1s} = \alpha_{11}^n \hat{A}_{12}^* + \alpha_{22}^s \hat{A}_{22}^*,$$

$$(\alpha_{11}^n, \alpha_{22}^s) = \frac{1}{h} \int_{-h/2}^{h/2} (\alpha_{11}, \alpha_{22}) dz,$$

where

$$\bar{I}_1 = \frac{4f_n \cos \beta_t}{3 \sin \beta_t} + 2c_c - \frac{4f_n}{\tan \beta_t},$$

$$I_c = \frac{2f_n}{\sin \beta_t} + c_c - \frac{2f_n}{\tan \beta_t},$$

$$\bar{I}_2 = \frac{4f_n^3}{3 \sin \beta_t} + 2f_n^2 \left(c_c - \frac{2f_n}{\tan \beta_t} \right) \quad \text{for trapezoidal}$$

corrugation and $I_c = \pi r_n + 2d_n$, $c_c = 2r_n$, $\bar{I}_1 = \pi r_n$,

$$\bar{I}_2 = \frac{4d_n^3}{3} + 2\pi d_n^2 r_n + 8d_n r_n^2 + \pi r_n^3 \quad \text{for round}$$

corrugation, and the stiffness matrix components of the pre-corrugated shell panels are evaluated as

$$(\hat{A}_{ij}, \hat{D}_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z^2) dz, \quad (i, j = 1, 2, 6)$$

By applying a coordinate transformation procedure, the internal force resultants of the FG-GRMMC cylindrical shell can be expressed in the global coordinate system (Oxyz) as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* & 0 & 0 & 0 \\ A_{12}^* & A_{22}^* & A_{26}^* & 0 & 0 & 0 \\ A_{61}^* & A_{62}^* & A_{66}^* & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{11}^* & D_{12}^* & D_{16}^* \\ 0 & 0 & 0 & D_{12}^* & D_{22}^* & D_{26}^* \\ 0 & 0 & 0 & D_{61}^* & D_{62}^* & D_{66}^* \end{bmatrix} \times$$

$$\begin{bmatrix} \bar{\epsilon}_x^0 \\ \bar{\epsilon}_y^0 \\ \bar{\gamma}_{xy}^0 \\ -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{bmatrix} - \begin{bmatrix} \hat{\Phi}_{1x} \\ \hat{\Phi}_{1y} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta T, \quad (4)$$

where

$$A_{11}^* = \hat{A}_{11}^* \sin^4 \theta + 2(\hat{A}_{12}^* + 2\hat{A}_{66}^*) \sin^2 \theta \cos^2 \theta + \hat{A}_{22}^* \cos^4 \theta,$$

$$A_{12}^* = (\hat{A}_{11}^* + \hat{A}_{22}^* - 4\hat{A}_{66}^*) \sin^2 \theta \cos^2 \theta + \hat{A}_{12}^* (\sin^4 \theta + \cos^4 \theta),$$

$$A_{16}^* = (\hat{A}_{11}^* - \hat{A}_{12}^* - 2\hat{A}_{66}^*) \cos \theta \sin^3 \theta + (\hat{A}_{12}^* - \hat{A}_{22}^* + 2\hat{A}_{66}^*) \cos^3 \theta \sin \theta,$$

$$\begin{aligned}
A_{61}^* &= A_{16}^*, \\
A_{62}^* &= A_{26}^*, \\
A_{22}^* &= \hat{A}_{11}^* \cos^4 \theta + 2(\hat{A}_{12}^* + 2\hat{A}_{66}^*) \sin^2 \theta \cos^2 \theta \\
&\quad + \hat{A}_{22}^* \sin^4 \theta, \\
A_{26}^* &= (\hat{A}_{11}^* - \hat{A}_{12}^* - 2\hat{A}_{66}^*) \cos^3 \theta \sin \theta \\
&\quad + (\hat{A}_{12}^* - \hat{A}_{22}^* + 2\hat{A}_{66}^*) \cos \theta \sin^3 \theta, \\
A_{66}^* &= (\hat{A}_{11}^* + \hat{A}_{22}^* - 2\hat{A}_{12}^* - 2\hat{A}_{66}^*) \sin^2 \theta \cos^2 \theta \\
&\quad + \hat{A}_{66}^* (\sin^4 \theta + \cos^4 \theta), \\
D_{11}^* &= \hat{D}_{11}^* \sin^4 \theta + 2(\hat{D}_{12}^* + 2\hat{D}_{66}^*) \sin^2 \theta \cos^2 \theta \\
&\quad + \hat{D}_{22}^* \cos^4 \theta, \\
D_{12}^* &= (\hat{D}_{11}^* + \hat{D}_{22}^* - 4\hat{D}_{66}^*) \sin^2 \theta \cos^2 \theta \\
&\quad + \hat{D}_{12}^* (\sin^4 \theta + \cos^4 \theta), \\
D_{16}^* &= (\hat{D}_{11}^* - \hat{D}_{12}^* - 2\hat{D}_{66}^*) \cos \theta \sin^3 \theta \\
&\quad + (\hat{D}_{12}^* - \hat{D}_{22}^* + 2\hat{D}_{66}^*) \cos^3 \theta \sin \theta, \\
D_{22}^* &= \hat{D}_{11}^* \cos^4 \theta + 2(\hat{D}_{12}^* + 2\hat{D}_{66}^*) \sin^2 \theta \cos^2 \theta \\
&\quad + \hat{D}_{22}^* \sin^4 \theta, \\
D_{61}^* &= D_{16}^*, \\
D_{62}^* &= D_{26}^*, \\
D_{26}^* &= (\hat{D}_{11}^* - \hat{D}_{12}^* - 2\hat{D}_{66}^*) \cos^3 \theta \sin \theta \\
&\quad + (\hat{D}_{12}^* - \hat{D}_{22}^* + 2\hat{D}_{66}^*) \cos \theta \sin^3 \theta, \\
D_{66}^* &= (\hat{D}_{11}^* + \hat{D}_{22}^* - 2\hat{D}_{12}^* - 2\hat{D}_{66}^*) \sin^2 \theta \cos^2 \theta \\
&\quad + \hat{D}_{66}^* (\sin^4 \theta + \cos^4 \theta), \\
\hat{\Phi}_{1x} &= \hat{\Phi}_{1n} \sin^2 \theta + \hat{\Phi}_{1s} \cos^2 \theta, \\
\hat{\Phi}_{1y} &= \hat{\Phi}_{1n} \cos^2 \theta + \hat{\Phi}_{1s} \sin^2 \theta.
\end{aligned}$$

3. Governing formulations

To describe the nonlinear stability behavior of FG-GRMMC cylindrical shells under external pressure, the formulation is developed within the framework of Donnell-type shell theory incorporating von Kármán geometric nonlinearity. The kinematic relations are established by considering the deformation of an arbitrary point located at a distance from the shell mid-surface. Accordingly, the strain components across the shell thickness can be represented as follows [32]

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \bar{\varepsilon}_x^0 \\ \bar{\varepsilon}_y^0 \\ \bar{\gamma}_{xy}^0 \end{bmatrix} - Z \begin{bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{bmatrix}, \quad (5)$$

in which $\bar{\varepsilon}_x^0$ and $\bar{\varepsilon}_y^0$ correspond to the membrane normal strains, whereas $\bar{\gamma}_{xy}^0$ refers to the in-plane shear strain defined at the shell mid-surface, and

$$\begin{bmatrix} \bar{\varepsilon}_x^0 \\ \bar{\varepsilon}_y^0 \\ \bar{\gamma}_{xy}^0 \end{bmatrix} = \begin{bmatrix} u_{,x} + w_{,x}^2/2 \\ -w/R + v_{,y} + w_{,y}^2/2 \\ v_{,x} + w_{,x}w_{,y} + u_{,y} \end{bmatrix}, \quad (6)$$

where $u = u(x, y)$, $v = v(x, y)$, and $w = w(x, y)$ denote the displacement components in the axial, circumferential, and transverse directions, respectively.

Based on Eq. (6), the compatibility condition for the shell strains can be expressed as follows

$$\bar{\varepsilon}_{y,xx}^0 - \bar{\gamma}_{xy,xy}^0 + \bar{\varepsilon}_{x,yy}^0 = w_{,xy}^2 - w_{,xx}w_{,yy} - \frac{1}{R}w_{,xx}. \quad (7)$$

To automatically satisfy the equilibrium conditions, the Airy stress function is introduced in the following form

$$N_x = \bar{\varphi}_{,yy}, \quad N_y = \bar{\varphi}_{,xx}, \quad N_{xy} = -\bar{\varphi}_{,xy}. \quad (8)$$

By substituting Eq. (4) into Eq. (7), considering Eq. (8), the latter can be reformulated as follows

$$\begin{aligned}
&\bar{A}_{22}^* \bar{\varphi}_{,xxxx} - (\bar{A}_{26}^* + \bar{A}_{62}^*) \bar{\varphi}_{,xxxy} - (\bar{A}_{16}^* + \bar{A}_{61}^*) \bar{\varphi}_{,xyyy} \\
&+ (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) \bar{\varphi}_{,xxyy} + \bar{A}_{11}^* \bar{\varphi}_{,yyyy} - w_{,xy}^2 \\
&+ w_{,xx}w_{,yy} + \frac{1}{R}w_{,xx} = 0.
\end{aligned} \quad (9)$$

where

$$\begin{aligned}
\Delta &= A_{11}^* A_{22}^* A_{66}^* - A_{11}^* A_{26}^* A_{62}^* - A_{12}^{*2} A_{66}^* + A_{12}^* A_{26}^* A_{61}^* \\
&\quad + A_{16}^* A_{12}^* A_{62}^* - A_{16}^* A_{22}^* A_{61}^*, \\
\bar{A}_{11}^* &= \frac{A_{22}^* A_{66}^* - A_{26}^* A_{62}^*}{\Delta}, \quad \bar{A}_{12}^* = -\frac{A_{12}^* A_{66}^* - A_{16}^* A_{62}^*}{\Delta}, \\
\bar{A}_{16}^* &= \frac{A_{12}^* A_{26}^* - A_{16}^* A_{22}^*}{\Delta}, \quad \bar{A}_{21}^* = -\frac{A_{12}^* A_{66}^* - A_{26}^* A_{61}^*}{\Delta}, \\
\bar{A}_{22}^* &= \frac{A_{11}^* A_{66}^* - A_{16}^* A_{61}^*}{\Delta}, \quad \bar{A}_{26}^* = \frac{A_{11}^* A_{26}^* - A_{16}^* A_{12}^*}{\Delta},
\end{aligned}$$

$$\bar{A}_{61}^* = \frac{A_{12}^* A_{62}^* - A_{61}^* A_{22}^*}{\Delta}, \quad \bar{A}_{62}^* = \frac{A_{11}^* A_{62}^* - A_{61}^* A_{12}^*}{\Delta},$$

$$\bar{A}_{66}^* = \frac{A_{11}^* A_{22}^* - A_{12}^{*2}}{\Delta}.$$

4. Assumed Deflection Function and Solution Procedure

An FG-GRMMC cylindrical shell under external pressure is considered, with simply supported boundary conditions at both ends, expressed as

$$\text{At } x = 0, L: w = N_{xy} = N_x = M_x = 0. \quad (10)$$

Within the Ritz framework, an approximate form of the transverse displacement is assumed as follows [27]

$$w = \bar{W}_0 + \bar{W}_1 \sin a_m x \sin b_n y + \bar{W}_2 \sin^2 a_m x \quad (11)$$

$$\text{where } a_m = \frac{m\pi}{L}; b_n = \frac{n}{R}.$$

Upon substituting the deflection and imperfection functions in Eq. (11) into Eq. (9), the Airy stress function is derived in the following form

$$\bar{\varphi} = \bar{\varphi}_1 \cos 2a_m x + \bar{\varphi}_2 \cos 2b_n y - \sigma_{0y} h \frac{x^2}{2}$$

$$+ \bar{\varphi}_3 \sin a_m x \sin b_n y + \bar{\varphi}_4 \sin 3a_m x \sin b_n y$$

$$+ \bar{\varphi}_5 \cos a_m x \cos b_n y + \bar{\varphi}_6 \cos 3a_m x \cos b_n y, \quad (12)$$

where

$$\bar{\varphi}_1 = k_1 \bar{W}_2 + k_2 \bar{W}_1^2, \bar{\varphi}_2 = k_3 \bar{W}_1^2,$$

$$\bar{\varphi}_3 = k_4 \bar{W}_1 \bar{W}_2 + k_5 \bar{W}_1, \bar{\varphi}_4 = k_6 \bar{W}_1 \bar{W}_2,$$

$$\bar{\varphi}_5 = k_7 \bar{W}_1 \bar{W}_2 + k_8 \bar{W}_1, \bar{\varphi}_6 = k_9 \bar{W}_1 \bar{W}_2,$$

$$k_1 = -\frac{1}{8Ra_m^2 \bar{A}_{22}^*}, \quad k_2 = \frac{b_n^2}{32a_m^2 \bar{A}_{22}^*}, \quad k_3 = \frac{a_m^2}{32b_n^2 \bar{A}_{11}^*},$$

$$k_0 = \begin{bmatrix} a_m^4 \bar{A}_{22}^* - a_m^3 b_n (\bar{A}_{26}^* + \bar{A}_{62}^*) \\ + a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) \\ - a_n b_n^3 (\bar{A}_{16}^* + \bar{A}_{61}^*) + b_n^4 \bar{A}_{11}^* \end{bmatrix}$$

$$\times \begin{bmatrix} -a_m^4 \bar{A}_{22}^* - a_m^3 b_n (\bar{A}_{26}^* + \bar{A}_{62}^*) \\ - a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) \\ - a_n b_n^3 (\bar{A}_{16}^* + \bar{A}_{61}^*) - b_n^4 \bar{A}_{11}^* \end{bmatrix},$$

$$k_4 = \frac{\begin{bmatrix} a_m^4 \bar{A}_{22}^* + a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) \\ + b_n^4 \bar{A}_{11}^* \end{bmatrix} a_m^2 b_n^2}{k_0},$$

$$k_5 = -\frac{\begin{bmatrix} a_m^4 \bar{A}_{22}^* + a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) \\ + b_n^4 \bar{A}_{11}^* \end{bmatrix} a_m^2}{Rk_0},$$

$$k_0^* = \begin{bmatrix} 81a_m^4 \bar{A}_{22}^* - 27a_m^3 b_n (\bar{A}_{26}^* + \bar{A}_{62}^*) + b_n^4 \bar{A}_{11}^* \\ + 9a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) - 3a_n b_n^3 (\bar{A}_{16}^* + \bar{A}_{61}^*) \end{bmatrix}$$

$$\times \begin{bmatrix} 81a_m^4 \bar{A}_{22}^* + 27a_m^3 b_n (\bar{A}_{26}^* + \bar{A}_{62}^*) - b_n^4 \bar{A}_{11}^* \\ + 9a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) + 3a_n b_n^3 (\bar{A}_{16}^* + \bar{A}_{61}^*) \end{bmatrix},$$

$$k_6 = \frac{\begin{bmatrix} 81a_m^4 \bar{A}_{22}^* + 9a_m^2 b_n^2 (\bar{A}_{66}^* + \bar{A}_{12}^* + \bar{A}_{21}^*) \\ + b_n^4 \bar{A}_{11}^* \end{bmatrix} b_n^2 a_m^2}{k_0^*},$$

$$k_7 = -\frac{\begin{bmatrix} a_m^2 (\bar{A}_{26}^* + \bar{A}_{62}^*) + b_n^2 (\bar{A}_{16}^* + \bar{A}_{61}^*) \end{bmatrix} a_m^3 b_n^3}{k_0},$$

$$k_8 = \frac{\begin{bmatrix} a_m^2 (\bar{A}_{26}^* + \bar{A}_{62}^*) + b_n^2 (\bar{A}_{16}^* + \bar{A}_{61}^*) \end{bmatrix} \frac{a_m^3 b_n}{R}}{k_0},$$

$$k_9 = -\frac{3a_m^3 b_n^3 \left[9a_m^2 (\bar{A}_{26}^* + \bar{A}_{62}^*) + b_n^2 (\bar{A}_{16}^* + \bar{A}_{61}^*) \right]}{k_0^*}.$$

Moreover, the shell closure condition can be represented in an averaged sense as follows

$$\int_0^{2\pi R} \int_0^L v_{,y} dx dy = \int_0^{2\pi R} \int_0^L \left(\bar{\varepsilon}_y^0 + \frac{w}{R} - \frac{w_{,y}^2}{2} \right) dx dy = 0 \quad (13)$$

After some algebraic manipulations, the circumferential stress can be obtained from Eq. (13) as follows

$$\sigma_{0y} = \frac{\bar{W}_2}{2Rh\bar{A}_{22}^*} - \frac{b_n^2 \bar{W}_1^2}{8h\bar{A}_{22}^*} + \frac{\bar{W}_0}{Rh\bar{A}_{22}^*}$$

$$+ \frac{(\hat{\Phi}_{1x} \bar{A}_{21}^* + \hat{\Phi}_{1y} \bar{A}_{22}^*) \Delta T}{h\bar{A}_{22}^*}, \quad (14)$$

The total potential energy of the shell can be formulated as follows

$$\bar{\Delta}_{\text{Total}} = \bar{\Delta}_{\text{in}} - \bar{\Delta}_{\text{ext}}. \quad (15)$$

The elastic strain energy of the shell is

evaluated as follows

$$\bar{\Delta}_{in} = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^{2\pi R} \int_0^L \left(\sigma_{xy} \gamma_{xy} + \sigma_x \varepsilon_x + \sigma_y \varepsilon_y \right) dx dy dz. \quad (16)$$

The potential of the external pressure load is given by

$$\bar{\Delta}_{ext} = \int_0^{2\pi R} \int_0^L q w dx dy. \quad (17)$$

Note that: Since the present study focuses on global buckling behavior, the external pressure is assumed to be uniformly distributed over the equivalent shell surface.

By applying the Ritz energy method, the governing equations can be obtained as follows

$$\frac{\partial \bar{\Delta}_{Total}}{\partial \bar{W}_0} = 0, \quad \frac{\partial \bar{\Delta}_{Total}}{\partial \bar{W}_1} = 0, \quad \frac{\partial \bar{\Delta}_{Total}}{\partial \bar{W}_2} = 0. \quad (18)$$

By combining Eqs. (14) and (18), the equilibrium equations of the shell can be obtained in the following form

$$H_{11} \bar{W}_0 + H_{12} \bar{W}_1^2 + H_{13} \bar{W}_2 - 2q + H_{15} \Delta T = 0 \quad (19)$$

$$H_{21} \bar{W}_0 + H_{22} \bar{W}_1^2 + H_{23} \bar{W}_2 + H_{24} \bar{W}_2^2 + H_{25} + H_{27} \Delta T = 0 \quad (20)$$

$$H_{31} \bar{W}_0 + H_{32} \bar{W}_2 \bar{W}_1^2 + H_{33} \bar{W}_1^2 + H_{34} \bar{W}_2 - q + H_{37} \Delta T = 0 \quad (21)$$

where

$$H_{11} = \frac{2}{R^2 \bar{A}_{22}^*}, \quad H_{12} = -\frac{b_n^2}{4R \bar{A}_{22}^*}, \quad H_{13} = \frac{1}{R^2 \bar{A}_{22}^*},$$

$$H_{15} = \frac{2\hat{\Phi}_{1y} \bar{A}_{22}^* + (\bar{A}_{12}^* + \bar{A}_{21}^*) \hat{\Phi}_{1x}}{R \bar{A}_{22}^*}, \quad H_{21} = -\frac{b_n^2}{2R \bar{A}_{22}^*},$$

$$H_{22} = \frac{64k_2^2 a_m^4 \bar{A}_{22}^{*2} + 64\bar{A}_{11}^* k_3^2 b_n^4 \bar{A}_{22}^* + b_n^4/8}{2\bar{A}_{22}^*},$$

$$H_{23} = 32k_2 k_1 a_m^4 \bar{A}_{22}^* - \frac{b_n^2}{4R \bar{A}_{22}^*}$$

$$+ (k_8 k_7 + k_5 k_4) \left[a_m^4 \bar{A}_{22}^* + \bar{A}_{11}^* b_n^4 + (\bar{A}_{16}^* + \bar{A}_{61}^*) a_m b_n^3 \right] + (\bar{A}_{12}^* + \bar{A}_{21}^* + \bar{A}_{66}^*) a_m^2 b_n^2 + (\bar{A}_{62}^* + \bar{A}_{26}^*) a_m^3 b_n$$

$$H_{24} = \frac{1}{2} \left[\begin{aligned} & (81k_6^2 + k_7^2 + 81k_9^2 + k_4^2) a_m^4 \bar{A}_{22}^* \\ & + \bar{A}_{11}^* (k_4^2 + k_6^2 + k_7^2 + k_9^2) b_n^4 \\ & + (6k_6 k_9 + 2k_7 k_4) (\bar{A}_{16}^* + \bar{A}_{61}^*) a_m b_n^3 \\ & + (\bar{A}_{12}^* + \bar{A}_{21}^* + \bar{A}_{66}^*) \left(\frac{9k_6^2 + k_7^2 + 9k_9^2}{+k_4^2} \right) a_m^2 b_n^2 \\ & + (54k_6 k_9 + 2k_7 k_4) (\bar{A}_{62}^* + \bar{A}_{26}^*) a_m^3 b_n \end{aligned} \right],$$

$$H_{25} = \frac{1}{2} \left[\begin{aligned} & \bar{A}_{22}^* (k_5^2 + k_8^2) - D_{11}^* \right] a_m^4 \\ & + (\bar{A}_{62}^* + \bar{A}_{26}^*) k_5 k_8 a_m^3 b_n + k_5 k_8 (\bar{A}_{16}^* + \bar{A}_{61}^*) a_m b_n^3 \\ & + \frac{1}{2} \left[\bar{A}_{11}^* (k_5^2 + k_8^2) - D_{22}^* \right] b_n^4 \\ & + \frac{1}{2} \left[(\bar{A}_{12}^* + \bar{A}_{21}^* + \bar{A}_{66}^*) (k_5^2 + k_8^2) - D_{12}^* - D_{21}^* \right] b_n^2 a_m^2, \end{aligned} \right]$$

$$H_{32} = \frac{1}{2} a_m^4 (k_4^2 + 81k_6^2 + k_7^2 + 81k_9^2) \bar{A}_{22}^* + (27k_6 k_9 + k_7 k_4) (\bar{A}_{62}^* + \bar{A}_{26}^*) a_m^3 b_n + \frac{1}{2} (\bar{A}_{12}^* + \bar{A}_{21}^* + \bar{A}_{66}^*) (9k_6^2 + k_7^2 + 9k_9^2 + k_4^2) a_m^2 b_n^2 + (3k_6 k_9 + k_7 k_4) (\bar{A}_{16}^* + \bar{A}_{61}^*) a_m b_n^3 + \frac{1}{2} \bar{A}_{22}^* (k_4^2 + k_6^2 + k_7^2 + k_9^2) b_n^4,$$

$$H_{27} = -\frac{[2\hat{\Phi}_{1y} \bar{A}_{22}^* + (\bar{A}_{12}^* + \bar{A}_{21}^*) \hat{\Phi}_{1x}] b_n^2}{4\bar{A}_{22}^*},$$

$$H_{31} = \frac{1}{R^2 \bar{A}_{22}^*},$$

$$H_{33} = a_m^4 \left(16k_2 k_1 + \frac{1}{2} k_5 k_4 + \frac{1}{2} k_8 k_7 \right) \bar{A}_{22}^* + \frac{1}{2} (\bar{A}_{62}^* + \bar{A}_{26}^*) (k_4 k_8 + k_7 k_5) a_m^3 b_n + \frac{1}{2} (\bar{A}_{12}^* + \bar{A}_{21}^* + \bar{A}_{66}^*) (k_5 k_4 + k_8 k_7) a_m^2 b_n^2 + \frac{1}{2} (\bar{A}_{16}^* + \bar{A}_{61}^*) (k_8 k_4 + k_7 k_5) a_m b_n^3 + \frac{1}{2} (k_5 k_4 + k_8 k_7) \bar{A}_{11}^* b_n^4 - \frac{b_n^2}{8R \bar{A}_{22}^*},$$

$$H_{34} = 16k_1^2 a_m^4 \bar{A}_{22}^* - 4a_m^4 D_{11}^* + \frac{1}{2R^2 \bar{A}_{22}^*},$$

$$H_{37} = \frac{2\hat{\Phi}_{1y} \bar{A}_{22}^* + \hat{\Phi}_{1x} (\bar{A}_{12}^* + \bar{A}_{21}^*)}{2R \bar{A}_{22}^*}.$$

By solving Eq. (19), the closed-form expression for $\bar{W}_0$ is obtained as follows

$$\bar{W}_0 = -\frac{H_{12}\bar{W}_1^2 + H_{13}\bar{W}_2 - 2q + H_{15}\Delta T}{H_{11}} \quad (22)$$

Upon substitution of Eq. (22) into Eq. (20), the closed-form expression for $\bar{W}_1$ is obtained as follows

$$\bar{W}_1^2 = T_{12}\bar{W}_2^2 + T_{11}\bar{W}_2 + T_{15}q + T_{16}\Delta T + T_{13} \quad (23)$$

This leads to an explicit relationship between the applied pressure and the deflection amplitude $\bar{W}_2$, as

$$q = -\frac{T_{21}\bar{W}_2^3 + T_{22}\bar{W}_2^2 + T_{23}\bar{W}_2 + T_{28} + (T_{29}\bar{W}_2 + T_{210})\Delta T}{T_{26}\bar{W}_2 + T_{27}} \quad (24)$$

where

$$\begin{aligned} T_{11} &= -\frac{H_{11}H_{23} - H_{13}H_{21}}{H_{11}H_{22} - H_{12}H_{21}}, \\ T_{12} &= -\frac{H_{24}H_{11}}{H_{11}H_{22} - H_{12}H_{21}}, \\ T_{13} &= -\frac{H_{25}H_{11}}{H_{11}H_{22} - H_{12}H_{21}}, \quad T_{15} = -\frac{2H_{21}}{H_{11}H_{22} - H_{12}H_{21}}, \\ T_{16} &= -\frac{H_{11}H_{27} - H_{15}H_{21}}{H_{11}H_{22} - H_{12}H_{21}}, \quad T_{21} = H_{32}T_{12}, \quad T_{26} = H_{32}T_{15}, \\ T_{22} &= -\frac{H_{31}H_{12}T_{12}}{H_{11}} + H_{32}T_{11} + H_{33}T_{12}, \\ T_{29} &= H_{32}T_{16}, \\ T_{23} &= \frac{H_{31}(-H_{12}T_{11} - H_{13})}{H_{11}} + T_{13}H_{32} + H_{33}T_{11} + H_{34}, \end{aligned}$$

$$T_{27} = \frac{H_{31}(-H_{12}T_{15} + 2)}{H_{11}} + T_{15}H_{33} - 1,$$

$$T_{28} = -\frac{H_{31}H_{12}T_{13}}{H_{11}} + T_{13}H_{33},$$

$$T_{210} = \frac{H_{31}(-H_{12}T_{16} - H_{15})}{H_{11}} + T_{16}H_{33} + H_{37}.$$

According to the bifurcation criterion, the upper critical buckling load is determined by taking the limit $\bar{W}_2 \rightarrow 0$, yielding:

$$q_{\text{upper}} = -\frac{T_{210}\Delta T + T_{28}}{T_{27}} \quad (25)$$

For numerical evaluation, the critical buckling load q_{cr} is taken as the lowest upper critical load over all admissible buckling modes (m, n) .

The total deflection amplitude, as defined in Eq. (11), can be evaluated as follows

$$W_{\text{max}} = \bar{W}_0 + \bar{W}_1 + \bar{W}_2 \quad (26)$$

Upon substitution of Eqs. (22) and (23) into Eq. (26), the expression for the total deflection amplitude is derived as follows

$$W_{\text{max}} = T_{31}\bar{W}_2^2 + T_{32}\bar{W}_2 + T_{34}q + T_{35} + T_{36}\Delta T + \sqrt{T_{12}\bar{W}_2^2 + T_{11}\bar{W}_2 + T_{15}q + T_{16}\Delta T + T_{13}} \quad (27)$$

where

$$\begin{aligned} T_{31} &= -\frac{H_{12}T_{12}}{H_{11}}, \quad T_{32} = \frac{-H_{12}T_{11} - H_{13}}{H_{11}} + 1, \\ T_{34} &= \frac{-H_{12}T_{15} + 2}{H_{11}}, \quad T_{35} = -\frac{H_{12}T_{13}}{H_{11}} + T_{13}, \\ T_{36} &= \frac{-H_{12}T_{16} - H_{15}}{H_{11}}. \end{aligned}$$

Table 1. Validation of critical buckling loads q_{upper} (in kN) of FG-GRC cylindrical shells under external pressure ($(0)_{10}$, $R/h = 20$, $h = 2\text{mm}$, $T = 300\text{K}$).

L^2/Rh	Shen and Xiang [22]		Present	
	UD	FG-X	UD	FG-X
100	20.231 (1;4) ^a	22.162 (1;4)	20.885 (1;4)	23.087 (1;4)
300	11.173 (1;3)	12.082 (1;3)	11.335 (1;3)	12.346 (1;3)
500	9.525 (1;3)	10.426 (1;3)	9.678 (1;3)	10.671 (1;3)

^a the buckling modes $(m; n)$.

Table 2. Critical buckling loads of non-corrugated and corrugated FG-GRMMC cylindrical shells (MPa, $h = 3\text{mm}$, $R = 80h$, $L = 1.5R$, $T = 500\text{K}$), trapezoidal corrugation ($f_n = 6\text{mm}$, $c_c = 4f_n$, $\beta_t = \pi/4$), round corrugation ($r_n = 5\text{mm}$, $d_n = 1\text{mm}$, $c_c = 2r_n$)

		UD	FG-X	FG-O
Trapezoidal corrugation	Helically corrugated	21.88 (1;3;28;63.03) ^b	22.16 (1;4;28;63.03)	21.26 (1;4;27;59.25)
	Ring corrugated	15.49 (1;3) ^a	15.73 (1;3)	15.03 (1;3)
	Non-corrugated	4.12 (1;6)	4.49 (1;6)	3.69 (1;6)
Round corrugation	Helically corrugated	25.08 (1;4;65;59.55)	25.16 (1;4;65;59.55)	24.72 (1;4;65;59.55)
	Ring corrugated	17.58 (1;3)	17.80 (1;3)	17.13 (1;3)
	Non-corrugated	11.05 (1;6)	12.24 (1;6)	9.70 (1;6)

^b the buckling modes ($m;n;N;\theta$)

Table 3. Effects of thermal environment on the critical buckling loads of corrugated FG-GRMMC cylindrical shells (MPa, FG-X, $h = 3\text{mm}$, $R = 80h$, $L = 1.5R$, trapezoidal corrugation ($f_n = 6\text{mm}$, $c_c = 4f_n$, $\beta_t = \pi/4$), round corrugation ($r_n = 5\text{mm}$, $d_n = 1\text{mm}$, $c_c = 2r_n$)

	T (K)	Ring corrugated	Helically corrugated
Trapezoidal corrugation	300	16.64 (1;3)	23.42 (1;4;28;63.03)
	500	15.73 (1;3)	22.16 (1;3;28;63.03)
	700	14.80 (1;3)	20.83 (1;4;28;63.03)
	1000	12.68 (1;3)	17.71 (1;4;28;63.03)
Round corrugation	300	18.83 (1;3)	26.62 (1;3;66;61.09)
	500	17.80 (1;3)	25.16 (1;4;65;59.55)
	700	16.75 (1;3)	23.69 (1;3;66;61.09)
	1000	14.32 (1;3)	20.31 (1;4;66;61.09)

5. Numerical Results and Parametric Discussion

Table 1 is used to validate the accuracy of the present analytical formulation through comparison with the results reported by Shen and Xiang [22] for FG-GRC cylindrical shells under external pressure. It can be observed that the present predictions agree well with the available reference data for both UD and FG-X distributions over different geometric ratios L^2/Rh . The discrepancies are small in all considered cases, which confirms the reliability of the proposed formulation.

Table 2 provides a comparative study on the

critical buckling loads of non-corrugated, ring-corrugated, and helically corrugated FG-GRMMC cylindrical shells under external pressure. It is clearly observed that the introduction of corrugation significantly enhances the load-carrying capacity of the shell, with the helically corrugated configuration exhibiting the highest buckling resistance among all cases. In particular, the critical buckling loads of helically corrugated shells are substantially higher than those of both ring-corrugated and non-corrugated shells. This improvement can be attributed to the unique geometric configuration of helical corrugation,

which induces a coupling between axial and circumferential stiffness components. As a result, the structure exhibits enhanced resistance against global instability under external pressure. In contrast, ring corrugation mainly reinforces the shell in the circumferential direction, leading to a more limited improvement in stability. Moreover, it is observed that the round corrugation profile generally yields higher critical loads compared to

the trapezoidal one.

The results also indicate that the graphene distribution pattern continues to play a significant role, with FG-X configurations consistently outperforming UD and FG-O distributions. This confirms that the optimal material gradation enhances the stiffness of the equivalent shell model, further contributing to stability improvement.

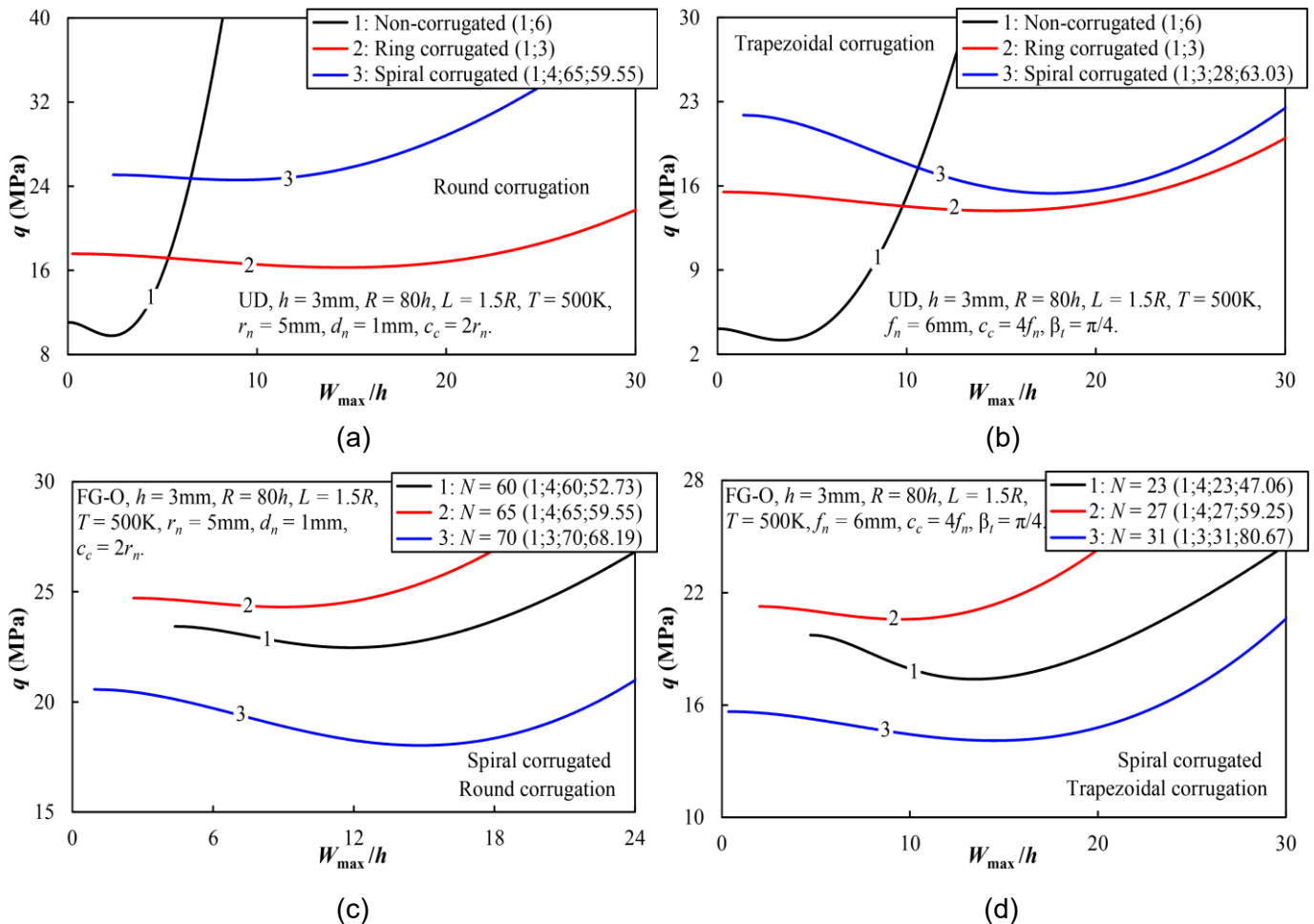


Fig. 2. Effects of corrugation types and the number of corrugation waves per strip on the postbuckling behavior. (a) Effects of corrugation types (round corrugation); (b) Effects of corrugation types (trapezoidal corrugation); (c) Effects of corrugation waves per strip (round corrugation); (d) Effects of corrugation waves per strip (trapezoidal corrugation);

Table 3 further examines the influence of temperature on the critical buckling behavior of corrugated FG-GRMMC cylindrical shells. It is clearly observed that the critical buckling load decreases significantly as the temperature increases for both ring-corrugated and helically corrugated configurations. This reduction in

buckling resistance can be attributed to two main factors. The increase in temperature leads to a degradation of the effective material stiffness of the FG-GRMMC, as the temperature-dependent elastic properties of the graphene-reinforced metal matrix decrease. Despite the adverse thermal effects, helically corrugated shells consistently

exhibit much higher critical loads compared to ring-corrugated shells across all temperature levels. This indicates that the beneficial stiffness enhancement induced by the helical geometry remains dominant even in elevated thermal environments. In other words, the coupling effect associated with helical corrugation provides a robust mechanism for maintaining structural stability under thermo-mechanical loading.

Fig. 2 presents the effects of corrugation type and the number of corrugation waves per strip on the postbuckling behavior of FG-GRMMC cylindrical shells under external pressure. In Figs. 2(a) and 2(b), the postbuckling curves of non-corrugated, ring-corrugated, and helically corrugated shells are compared for round and trapezoidal corrugation profiles, respectively. It is clearly seen that the non-corrugated shell exhibits the lowest buckling resistance and the most unfavorable postbuckling response, whereas both corrugated configurations lead to a significant improvement in stability. Among them, the helically corrugated shell consistently provides the highest pressure-carrying capacity over the entire postbuckling range. This result confirms that the introduction of corrugation not only raises the critical buckling load but also modifies the postbuckling path in a beneficial manner. In particular, the helical corrugation generates a stronger coupling between the axial and circumferential stiffness components, thereby improving the global resistance of the shell against external pressure. By contrast, the ring-corrugated shell mainly enhances the circumferential stiffness, so its stabilizing effect is less pronounced than that of the helical configuration. A similar trend is observed for both round and trapezoidal corrugations, although the round corrugation exhibits a slightly higher postbuckling level, indicating a more efficient stiffness distribution. Figs. 2(c) and 2(d) further illustrate the influence of the number of corrugation waves per strip, N , on the postbuckling response of helically corrugated

shells. It is observed that changing N does not merely shift the curves upward or downward, but also alters their shape and curvature. This indicates that N plays an important role in controlling the effective stiffness and nonlinear deformation pattern of the shell. For the round corrugation case in Fig. 2(c), increasing N from 60 to 65 leads to a noticeable enhancement in the postbuckling response, whereas a further increase to 70 reduces the load-carrying capacity in a wide deflection range. A similar non-monotonic tendency can also be identified for the trapezoidal corrugation in Fig. 2(d), where the intermediate value of N yields the most favorable response. These observations suggest that the number of corrugation waves per strip should be regarded as an optimization parameter rather than a quantity that monotonically improves stability.

Fig. 3 illustrates the effects of graphene distribution patterns, temperature, and shell geometric parameters on the postbuckling behavior of helically corrugated FG-GRMMC cylindrical shells. In general, all curves exhibit a nonlinear postbuckling response, while their relative positions clearly indicate the sensitivity of the shell stability to both material gradation and structural dimensions. Fig. 3(a) shows the influence of graphene distribution patterns for the round corrugation case. It is observed that the FG-X distribution yields the highest postbuckling path, followed by the UD configuration, whereas the FG-O distribution gives the lowest response. This trend indicates that the through-thickness distribution of graphene reinforcement has a pronounced effect on the effective stiffness of the shell. In the FG-X pattern, the reinforcement is arranged in a more favorable manner for resisting bending and membrane actions, thereby improving the global stability of the shell.

By contrast, the FG-O distribution provides a less efficient stiffness layout, resulting in lower pressure-carrying capacity throughout the postbuckling range. The effect of temperature is

presented in Fig. 3(b). As the temperature increases from 300K to 1000K, the entire postbuckling curve shifts downward, indicating a continuous reduction in both buckling resistance and postbuckling strength. This behavior is physically expected, since elevated temperature reduces the effective elastic properties of the FG-GRMMC and simultaneously introduces thermal membrane effects that promote instability. Figs.

3(c) and 3(d) depict the influence of shell length and shell radius, respectively. In Fig. 3(c), increasing the shell length leads to a marked reduction in the postbuckling load level. Longer shells are more slender and therefore more susceptible to global instability under external pressure. A similar tendency is observed in Fig. 3(d), where the load-carrying capacity decreases as the shell radius increases.

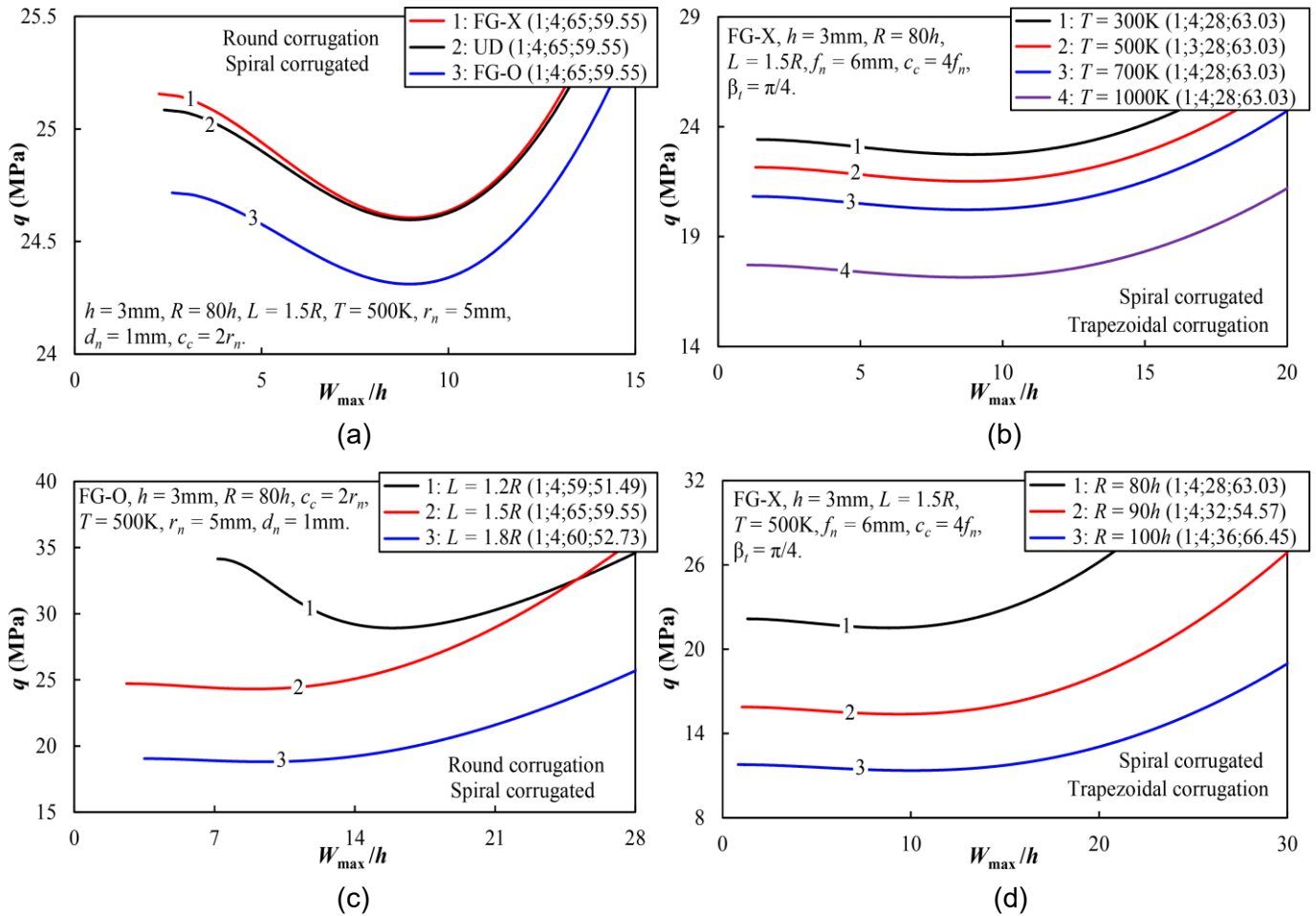


Fig. 3. Effects of graphene distributions, temperatures, and geometrical parameters of the shells on the postbuckling behavior. (a) Effects of graphene distributions; (b) Effects of temperatures; (c) Effects of the lengths of shells; (d) Effects of radii of shells

Fig. 4 presents the effects of corrugation geometric parameters on the postbuckling behavior of helically corrugated FG-GRMMC cylindrical shells. Unlike the global shell dimensions discussed in Fig. 3, the parameters considered here are directly associated with the local corrugation profile and therefore mainly influence the equivalent stiffness generated by the

homogenized corrugated structure. For the trapezoidal corrugation case, Fig. 4(a) shows that increasing the corrugation depth f_n leads to a clear enhancement of the postbuckling response. The curve corresponding to the largest value of f_n remains above the others over almost the entire deflection range, indicating a higher load-carrying capacity. Fig. 4(b) shows that the effect of the half-

wavelength parameter on the postbuckling response is non-monotonic. The case with the largest C_c generally exhibits the highest pressure level, although the curves intersect at certain deflection levels. This indicates that increasing the corrugation wavelength does not necessarily improve the postbuckling resistance. For the round corrugation profile, Fig. 4(c) shows the influence of the corrugation depth d_n . In this case, the postbuckling response also depends strongly on the corrugation geometry, although the effect is not

entirely monotonic over the full deflection range. At relatively small deflections, the shell with larger d_n exhibits a higher pressure level, while at larger deflections the curves approach and even intersect. Fig. 4(d) illustrates the effect of c_c for round corrugation. It is seen that increasing c_c causes a downward shift of the postbuckling curves, indicating a reduction in structural resistance. Therefore, a larger corrugation wavelength in the round profile does not improve stability in the present configuration.

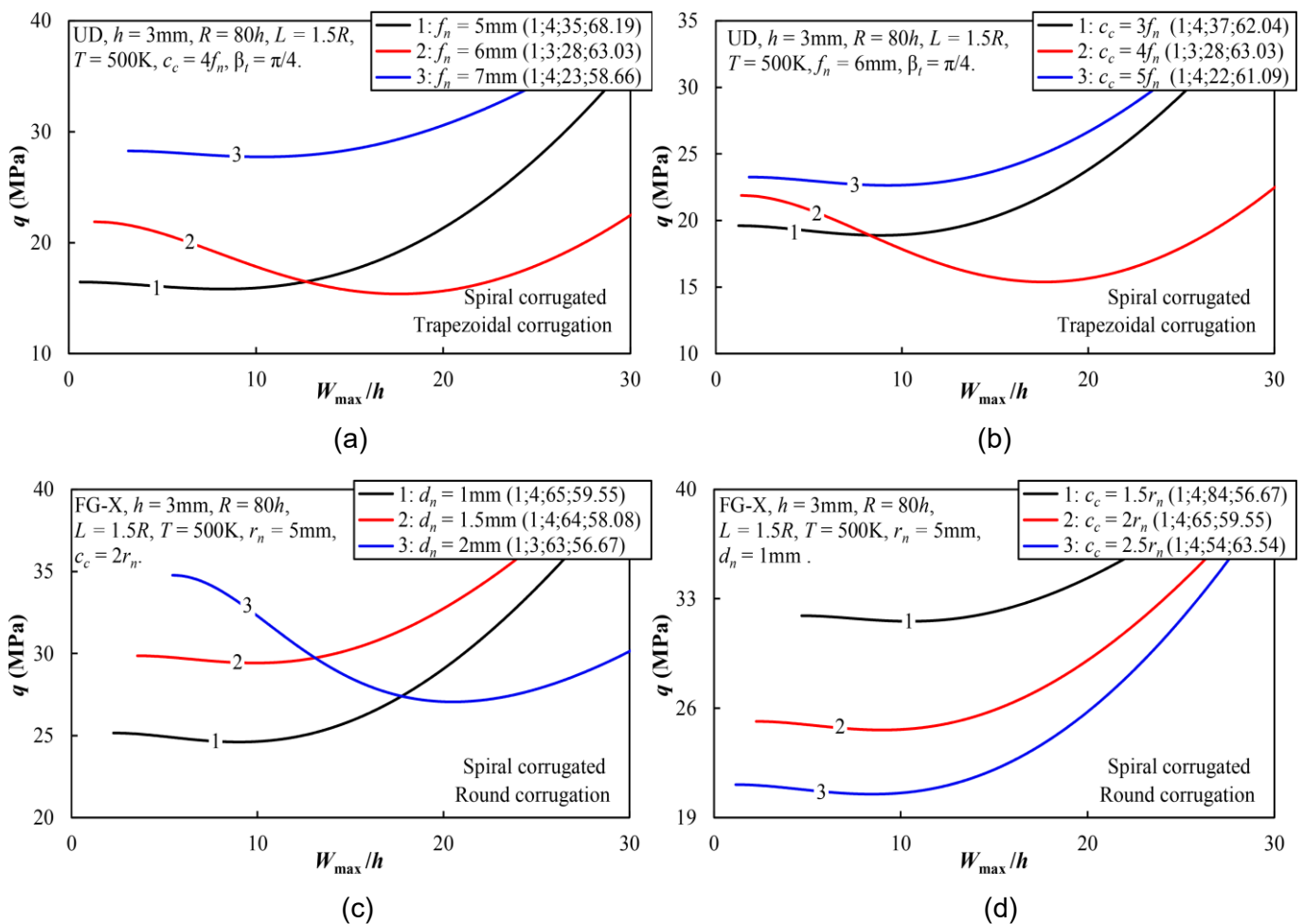


Fig. 4. Effects of geometrical parameters of corrugations on the postbuckling behavior. (a) Effects of f_n (trapezoidal corrugation); (b) Effects of c_c (trapezoidal corrugation); (c) Effects of d_n (round corrugation); (d) Effects of c_c (round corrugation)

6. Conclusion

This paper has presented an analytical formulation for the nonlinear buckling and postbuckling analysis of helically corrugated FG-

GRMMC cylindrical shells subjected to external pressure in a thermal environment. The formulation was developed on the basis of the Donnell shell theory with von Kármán geometric nonlinearity. An

equivalent-shell homogenization approach was employed to represent the corrugated geometry, while temperature-dependent material properties and effective thermal force resultants were incorporated into the thermo-mechanical constitutive relations. The nonlinear equilibrium equations were then solved by means of the Ritz energy method using a three-term assumed deflection function.

The numerical results show that the present formulation is in good agreement with available results in the literature, confirming the accuracy and reliability of the proposed approach. It was found that corrugation has a pronounced beneficial effect on the buckling behavior of FG-GRMMC cylindrical shells. In particular, helically corrugated shells exhibit considerably higher critical buckling loads and more favorable postbuckling responses than ring-corrugated and non-corrugated shells. This improvement is mainly attributed to the enhanced equivalent stiffness and the coupling effect induced by the helical corrugation geometry.

The parametric study further indicates that the structural response is strongly affected by temperature, graphene distribution pattern, shell geometry, and corrugation parameters. Increasing temperature leads to a significant reduction in both buckling resistance and postbuckling strength due to thermal softening of the material. Among the considered material distributions, the FG-X pattern provides the most favorable stability response. In addition, the number of corrugation waves per strip plays an important role in controlling the nonlinear behavior, and its effect is generally non-monotonic, implying the existence of an optimal corrugation configuration. The shell length and radius tend to reduce the load-carrying capacity when increased, whereas the local corrugation dimensions can either improve or deteriorate the postbuckling performance depending on the corrugation profile.

Declaration of Generative AI

During the preparation of this work, the authors do not use Generative AI and AI-assisted

technologies tools in order to generate the content of the manuscript.

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