

**Article info****Type of article:**

Original research paper

DOI:

<https://doi.org/10.58845/jstt.utt.2026.en.6.4.231-247>

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Email address:

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Received in Revised Form:
25/07/2026

Accepted: 05/08/2026

On the nonlinear vibration behavior of doubly curved FG-GRMMC shell panels with multilayer corrugated FG-GRMMC core

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Abstract: This study presents an analytical investigation into the nonlinear vibration characteristics of doubly curved sandwich panels composed of multilayer corrugated FG-GRMMC cores and two face sheets made from functionally graded graphene-reinforced metal matrix composite (FG-GRMMC). Harmonic excitation and a thermal environment are applied for the considered panels. An improved homogenization scheme is developed to represent the effective mechanical behavior of the layered corrugated core, accounting for both geometric configurations and graphene distribution patterns. The governing equations are established within the framework of Donnell shell theory, incorporating von Kármán geometric nonlinearity, and are reduced to a set of nonlinear ordinary differential equations using the Ritz energy approach. Both free and forced vibration responses are examined, including natural frequencies, nonlinear frequency–amplitude relationships, and dynamic responses. The effects of key parameters such as core geometry, graphene gradation, temperature variation, and shell curvature are systematically evaluated. The results demonstrate that the multilayer corrugated core significantly enhances the dynamic stiffness and vibration performance of the structure. The results offer valuable contributions into the design of sandwich shell structures subjected to complex thermomechanical and dynamic loading conditions.

Keywords: Nonlinear vibration, Ritz energy method, Corrugated core, Donnell shell theory, Functionally graded graphene reinforced metal matrix composite.

1. Introduction

Structural elements such as rectangular and spherical caps, cylindrical panels, and circular plates are extensively employed in advanced engineering applications. Consequently, a substantial body of research has focused on these configurations, addressing various mechanical problems including static and dynamic stability, as

well as vibrational behavior. To analyze these phenomena, numerous theoretical models, assumptions, and analytical approaches have been developed. Specifically, for functionally graded material (FGM) structures and sandwich FGM variants, encompassing rectangular and circular plates, cylindrical and spherical shells, a wide range of studies have been conducted on

their linear and nonlinear vibrations, postbuckling responses, dynamic instability, and static behavior [1-7]. These investigations have employed elasticity theory, Donnell shell theory, first-order and higher-order shear deformation theories (FSDT and HSDT), and five-degree-of-freedom shear deformable models. Analytical and semi-analytical methods such as the Ritz procedure, Chebyshev–Ritz approach, Galerkin method, perturbation techniques, Fourier series expansions, and the state-space differential quadrature method (DQM) have been utilized. Furthermore, both thermal and mechanical buckling characteristics of orthogonal and plates with oblique stiffeners, as well as porous-core FGM circular plates and spherical caps, have been examined using FSDT, HSDT, the Galerkin and Ritz methods, and the enhanced smeared stiffener technique [8, 9]. Linear free vibration, nonlinear free vibration, and nonlinear forced vibration of FGM doubly curved shallow shells were investigated using the power series expansion method of displacement components [10], the harmonic balance method [11], and the Galerkin method [12].

Functionally Graded Carbon Nanotube Reinforced Composite (FG-CNTRC) is an advanced composite material reinforced with carbon nanotubes, with properties that vary across its thickness, offering high mechanical strength, corrosion resistance, and flexibility. With excellent load-bearing capacity and thermal and electrical conductivity, FG-CNTRC is widely used in aerospace, automotive, and construction industries. Nonlinear thermal buckling behavior of FG-CNTRC rectangular plates employing the HSDT, perturbation technique, and multi-scale approach was investigated [13, 14]. By applying the element-free interpolating moving least-squares Ritz method [15], the two-step perturbation approach [16], and the Galerkin method [17], the static buckling, postbuckling, and nonlinear dynamic buckling analysis of FG-CNTRC

skew plates and cylindrical panels were studied. The free vibration, forced vibration, and wave propagation of FG-CNTRC and sandwich FG-CNTRC plates and doubly curved panels were mentioned with arbitrary shape [18], plate-foundation interaction [19], sensors and actuators [20], and with cracks [21].

The two-dimensional atomic configuration of graphene enables more effective interfacial bonding with the matrix, resulting in stronger interactions than those observed in carbon nanotube-reinforced systems, leading to improved load transfer efficiency. Within the field of advanced FGMs, polymer-based graphene-reinforced composites with functionally graded distributions (FG-GRC) have emerged as highly promising candidates for next-generation engineering applications [22-24], owing to these advantages. Using HSDT together with perturbation methods, numerous investigations have been carried out to examine the buckling, postbuckling, and vibration characteristics of FG-GRC plates [22, 23]. Using similar theoretical frameworks, the nonlinear vibration characteristics of FG-GRC cylindrical panels have also been investigated [24]. In addition, for FG-GRC plates, isogeometric finite element approaches have been used to study both nonlinear buckling [25] and free vibration behavior [26]. The linear buckling behavior of laminated FG-GRC plates with temperature-dependent properties has further been examined, including the influence of elastic foundation interactions [27]. The functionally graded graphene-reinforced metal matrix composite (FG-GRMMC) is an advanced composite material in which graphene reinforcements are distributed with a spatial gradient within a metallic matrix. Compared with FG-GRC materials, FG-GRMMC offers superior stiffness and thermal resistance due to its metal matrix, making it more suitable for applications involving high temperatures and demanding mechanical loading conditions. Shen et al. [28] and Shen and Xiang [29] investigated the effects of

negative Poisson('s) ratio on the postbuckling responses of FG-GRMMC plates and cylindrical panels using the two-step perturbation method and HSDT.

Widely employed in modern engineering, corrugated panels and shell systems made from advanced materials are valued for their capability to simultaneously achieve high load-bearing performance and reduced structural weight. Addressing supersonic flutter stability, Zheng et al. [30] formulated an analysis of sinusoidal and trapezoidal corrugated plates made from composite materials within the framework of Kirchhoff plate theory combined with supersonic piston theory. including uniaxial compression and in-plane shear, under different loading scenarios, for sinusoidally corrugated composite panels, the local buckling response was examined by Pathirana and Qiao [31, 32] through the application of Donnell-type shell theory together with the Rayleigh-Ritz approach. Beyond conventional materials, corrugated configurations have also been developed using functionally graded materials (FGMs). In particular, with the homogenization scheme originally proposed by Xia et al. [33], the interaction between internal fluid and corrugated sandwich FGM cylindrical shells under nonlinear free and forced vibrations was explored in [34], where Donnell shell theory was integrated. Distinct from traditional corrugated surfaces that mainly enhance external durability, corrugated cores serve as internal load-transferring frameworks, thereby improving structural efficiency and optimizing stress distribution. Such features render these configurations especially suitable for lightweight-critical applications, including aerospace and architectural systems. More recently, attention has been directed toward FG-GRC sandwich structures incorporating corrugated cores [35, 36]. Using a combination of the Ritz energy method, homogenization techniques [33], and Donnell shell theory, the mechanical responses of cylindrical shells and toroidal shell

segments under thermal loading conditions, such as axial tension/compression [35] and torsion [36], have been systematically evaluated. Furthermore, the homogenization approach has been extended to layered corrugated core systems composed of FG-CNTRC and FG-GRC materials, enabling detailed investigations of their nonlinear buckling characteristics, as reported in [37, 38].

Although significant research has been conducted on functionally graded nanocomposite panels and shells, most existing studies are limited to single-curvature geometries or flat structures and primarily focus on buckling, postbuckling, or linear vibration behaviors. Meanwhile, corrugated core sandwich structures have been widely investigated, but mainly in the form of single-layer cores or simplified equivalent models. The nonlinear dynamic response of sandwich panels with multilayer corrugated cores, particularly under combined effects of double curvature, FG-GRMMC, and thermal environment, remains largely unexplored. Therefore, a comprehensive analytical framework for such complex systems is still lacking.

Motivated by the above considerations, the present study develops an analytical approach to study the nonlinear vibration responses of doubly curved sandwich panels with multilayer corrugated cores made of FG-GRMMC. An improved homogenization approach is employed to characterize the effective properties of the layered corrugated core; based on nonlinearly large-deflection Donnell shell theory, the governing equations are formulated and then resolved through the application of the Ritz method. The model is then applied to examine free and forced nonlinear vibration responses under harmonic excitation in thermal environments, with particular attention to the effects of core geometry, graphene distribution, curvature, and temperature. The results are expected to provide useful insights for the design of advanced sandwich shell structures with complex core architectures under dynamic

loading conditions.

2. Design of FG-GRMMC shell panels with corrugated FG-GRMMC core

The present study considers a shell structure consisting of three orthogonally arranged corrugated FG-GRMMC core layers sandwiched

between two FG-GRMMC face sheets. An external pressure q is applied to the shell in a thermal environment T , with the geometry defined in the coordinate system. Both round and trapezoidal corrugation patterns are taken into account in the analysis, as shown in Fig. 1.

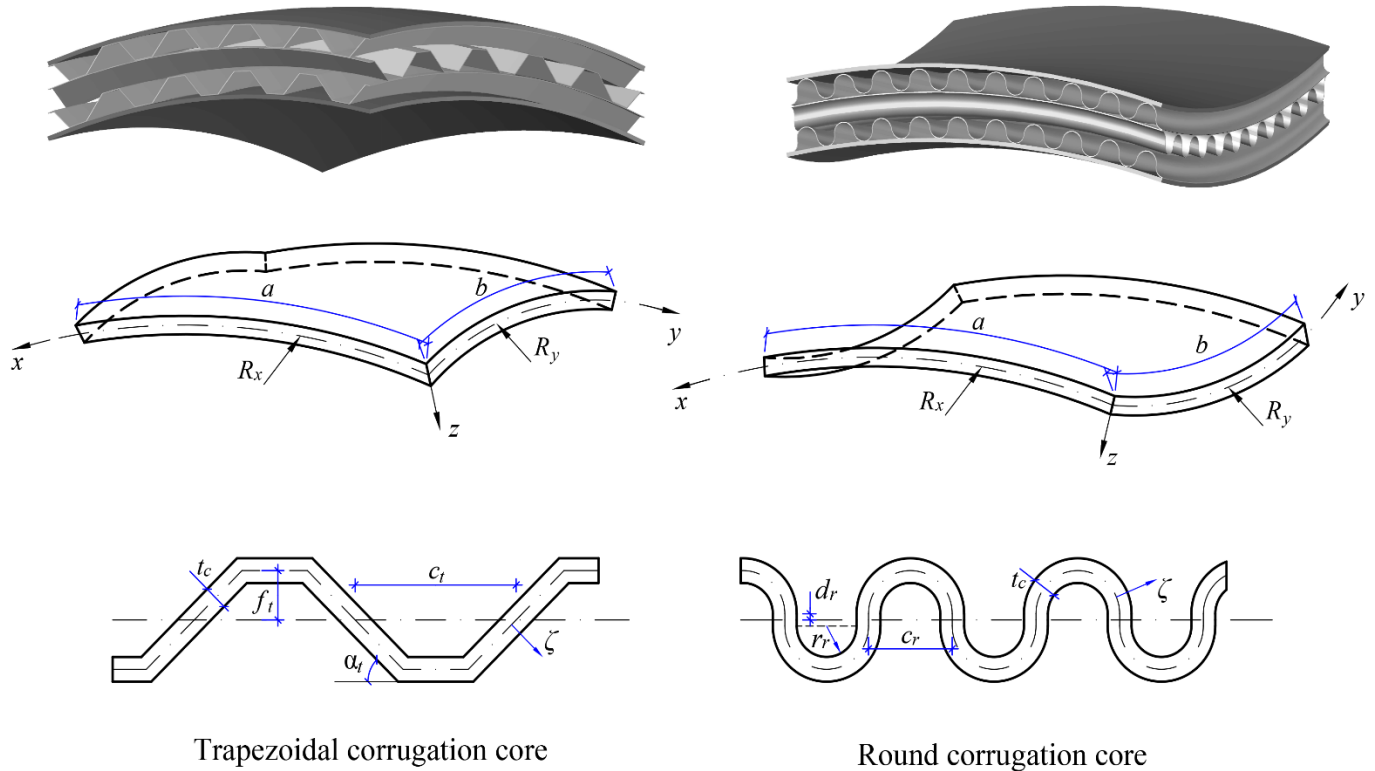


Fig. 1. Configurations of doubly curved FG-GRMMC shell panels with corrugated FG-GRMMC core and coordinate system

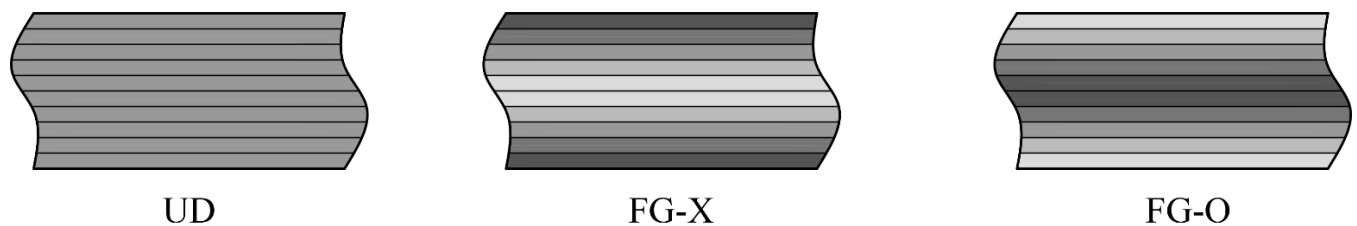


Fig. 2. Through-thickness distribution of graphene in the corrugated cores and face sheets

Two FG-GRMMC face sheets are designed with the same thicknesses h_f , meanwhile, the same thicknesses of three corrugated layers h_c are also chosen, with the corrugation wall thickness is denoted by t_c , and h is the total shell thickness of the panels.

The parameter c represents the half-length of corrugated waves, while r_r denotes the radius of these arcs. For the trapezoidal corrugation, the

half-amplitude is defined as f_t , and the corresponding trough angle is given by α_t . In addition, d_r is the half-period of the round corrugation.

A ten-layer piecewise gradation scheme is adopted to describe the variation of graphene content through the thickness of the FG-GRMMC shell. The reinforcement phase consists of graphene dispersed in a copper (Cu) matrix, with

five prescribed volume fractions: 0.05, 0.07, 0.09, 0.11, and 0.13. In the present study, two symmetric non-uniform graphene distribution patterns (FG-O and FG-X) are considered together with one uniform reference case (UD) (see Fig. 2). Specifically, the FG-O pattern is defined by $[(0.05)/(0.07)/(0.09)/(0.11)/(0.13)]_s$, whereas the FG-X pattern follows the reversed arrangement $[(0.13)/(0.11)/(0.09)/(0.07)/(0.05)]_s$. For the UD configuration, the graphene volume fraction is fixed at 0.09 for all ten layers. The temperature-dependent thermoelastic properties of the GRMMC constituents are taken from the works of Shen et al. [28, 29].

The strain field across the shell thickness, evaluated at a distance z from the mid-surface, can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} - z \begin{Bmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{Bmatrix} \quad (1)$$

Based on the nonlinear large-deflection strain-displacement relations for shells, the mid-surface strain components can be expressed as [12]

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \frac{1}{2}w_{,x}^2 + w_{,x}w_{,x}^* - w/R_x \\ -w/R_y + v_{,y} + w_{,y}w_{,y}^* + \frac{1}{2}w_{,y}^2 \\ w_{,x}w_{,y} + u_{,y} + v_{,x} + w_{,x}w_{,y}^* + w_{,y}w_{,x}^* \end{Bmatrix} \quad (2)$$

in which $u = u(x, y)$, $v = v(x, y)$ and $w = w(x, y)$ are the displacement components along x , y , and z axes, respectively.

Hooke's law for FG-GRMMC layers, accounting for thermal effects, can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \Delta T \begin{bmatrix} \alpha_{11} \\ \alpha_{22} \\ 0 \end{bmatrix}, \quad (3)$$

where the reduced stiffnesses Q_{ij} can be

expressed by

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{21}E_{11}}{1 - \nu_{21}\nu_{12}},$$

$$Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, \quad Q_{66} = G_{12}.$$

Building upon the homogenization approach proposed by Xia et al. [33], an enhanced formulation is developed to describe the mechanical behavior of FG-GRMMC sandwich shells with corrugated cores. By incorporating the effects of thermal loads and multilayer corrugated configurations, the resulting force and moment components can be expressed in an equivalent orthotropic form as follows

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{H}_{11} & \bar{H}_{12} & 0 & 0 & 0 & 0 \\ \bar{H}_{12} & \bar{H}_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{H}_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{S}_{11} & \bar{S}_{12} & 0 \\ 0 & 0 & 0 & \bar{S}_{12} & \bar{S}_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{S}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{Bmatrix} - \begin{Bmatrix} \bar{\Phi}_{1x} \\ \bar{\Phi}_{1y} \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (4)$$

where $\bar{H}_{ij}$ and $\bar{S}_{ij}$ are the stiffnesses of the FG-GRMMC panels, which are determined by summing those of three corrugated FG-GRMMC cores and two FG-GRMMC face sheets, presented as

$$\begin{aligned} \bar{H}_{ij} &= \bar{H}_{ij}^{tf} + \bar{H}_{ij}^{c(1)} + \bar{H}_{ij}^{c(2)} + \bar{H}_{ij}^{c(3)} + \bar{H}_{ij}^{bf}, \\ \bar{S}_{ij} &= \bar{S}_{ij}^{tf} + \bar{S}_{ij}^{c(1)} + \bar{S}_{ij}^{c(2)} + \bar{S}_{ij}^{c(3)} + \bar{S}_{ij}^{bf}, \\ \bar{\Phi}_{1x} &= \bar{\Phi}_{1x}^{tf} + \bar{\Phi}_{1x}^{c(1)} + \bar{\Phi}_{1x}^{c(2)} + \bar{\Phi}_{1x}^{c(3)} + \bar{\Phi}_{1x}^{bf}, \\ \bar{\Phi}_{1y} &= \bar{\Phi}_{1y}^{tf} + \bar{\Phi}_{1y}^{c(1)} + \bar{\Phi}_{1y}^{c(2)} + \bar{\Phi}_{1y}^{c(3)} + \bar{\Phi}_{1y}^{bf}, \end{aligned} \quad (5)$$

for the face sheets

$$\begin{aligned}
(\bar{H}_{ij}^{tf}, \bar{S}_{ij}^{tf}) &= \sum_{k=1}^{N_{tf}} \int_{\Gamma^{(k)}} Q_{ij}^{(k)}(1, z^2) dz, \\
(\bar{H}_{ij}^{bf}, \bar{S}_{ij}^{bf}) &= \sum_{k=1}^{N_{bf}} \int_{\Gamma^{(k)}} Q_{ij}^{(k)}(1, z^2) dz, \quad (i, j) = (1, 2, 6), \\
\bar{\Phi}_{1x}^{tf} &= \sum_{k=1}^{N_{tf}} \int_{\Gamma^{(k)}} [Q_{11}^{(k)} \alpha_{11}^{(k)} + Q_{12}^{(k)} \alpha_{22}^{(k)}] dz, \\
\bar{\Phi}_{1x}^{bf} &= \sum_{k=1}^{N_{bf}} \int_{\Gamma^{(k)}} [Q_{11}^{(k)} \alpha_{11}^{(k)} + Q_{12}^{(k)} \alpha_{22}^{(k)}] dz, \\
\bar{\Phi}_{1y}^{tf} &= \sum_{k=1}^{N_{tf}} \int_{\Gamma^{(k)}} [Q_{12}^{(k)} \alpha_{11}^{(k)} + Q_{22}^{(k)} \alpha_{22}^{(k)}] dz, \\
\bar{\Phi}_{1y}^{bf} &= \sum_{k=1}^{N_{bf}} \int_{\Gamma^{(k)}} [Q_{12}^{(k)} \alpha_{11}^{(k)} + Q_{22}^{(k)} \alpha_{22}^{(k)}] dz,
\end{aligned}$$

Where, for the face sheets, the number of layers is N_{tf} and N_{bf} , the k -th layer of the face sheets is designed by the thickness domain $\Gamma^{(k)}$. For the core, $\bar{H}_{ij}^{c(1)}, \bar{H}_{ij}^{c(2)}, \bar{H}_{ij}^{c(3)}, \bar{S}_{ij}^{c(1)}, \bar{S}_{ij}^{c(2)}, \bar{S}_{ij}^{c(3)}$ are the stiffnesses, and $\bar{\Phi}_{1x}^{c(1)}, \bar{\Phi}_{1x}^{c(2)}, \bar{\Phi}_{1x}^{c(3)}, \bar{\Phi}_{1y}^{c(1)}, \bar{\Phi}_{1y}^{c(2)}, \bar{\Phi}_{1y}^{c(3)}$ are the thermal forces, expressed as

- for x-direction core

$$\begin{aligned}
\bar{H}_{22}^{c(k^*)} &= \frac{2c}{\frac{l_1}{H_{11}} + \frac{l_2}{S_{11}}}, \quad \bar{H}_{12}^{c(k^*)} = \frac{\bar{H}_{22}^{c(k^*)} H_{12}}{H_{11}}, \\
\bar{H}_{11}^{c(k^*)} &= \frac{l_c}{c} \frac{H_{11} H_{22} - H_{12}^2}{H_{11}} + \frac{\bar{H}_{12}^{c(k^*)} H_{12}}{H_{11}}, \\
\bar{H}_{66}^{c(k^*)} &= \frac{\zeta}{l_c} H_{66}, \quad \bar{S}_{22}^{c(2)} = \frac{c}{l_c} S_{11}, \quad \bar{S}_{66}^{c(2)} = \frac{l_c}{c} S_{66}, \\
\bar{S}_{12}^{c(2)} &= \frac{S_{12}}{S_{11}} \bar{S}_{22}^{c(2)}, \quad \bar{S}_{11}^{c(2)} = \frac{l_c}{2c} (l_2 H_{22} + l_1 S_{22}), \\
\bar{S}_{ij}^{c(1)} &= \int_{-h_c/2}^{-h_c/6} Q_{ij}^{*c} z^2 dz, \quad \bar{S}_{ij}^{c(3)} = \int_{h_c/6}^{h_c/2} Q_{ij}^{*c} z^2 dz, \\
\bar{\Phi}_{1x}^{c(k^*)} &= \bar{H}_{11}^{c(k^*)} \bar{\alpha}_{11}^c + \bar{H}_{12}^{c(k^*)} \bar{\alpha}_{22}^c, \\
\bar{\Phi}_{1y}^{c(k^*)} &= \bar{H}_{12}^{c(k^*)} \bar{\alpha}_{11}^c + \bar{H}_{22}^{c(k^*)} \bar{\alpha}_{22}^c.
\end{aligned} \tag{6}$$

- for y-direction core

$$\bar{H}_{11}^{c(k^*)} = \frac{2c}{\frac{l_1}{H_{11}} + \frac{l_2}{S_{11}}}, \quad \bar{H}_{12}^{c(k^*)} = \frac{\bar{H}_{11}^{c(k^*)} H_{12}}{H_{11}},$$

$$\begin{aligned}
\bar{H}_{22}^{c(k^*)} &= \frac{l_c}{c} \frac{H_{11} H_{22} - H_{12}^2}{H_{11}} + \frac{\bar{H}_{12}^{c(k^*)} H_{12}}{H_{11}}, \\
\bar{H}_{66}^{c(k^*)} &= \frac{\zeta}{l_c} H_{66}, \quad \bar{S}_{11}^{c(2)} = \frac{c}{l_c} S_{11}, \\
\bar{S}_{12}^{c(2)} &= \frac{S_{12}}{S_{11}} \bar{S}_{11}^{c(2)}, \quad \bar{S}_{22}^{c(2)} = \frac{l_c}{2c} (l_2 H_{22} + l_1 S_{22}), \\
\bar{S}_{66}^{c(2)} &= \frac{l_c}{c} S_{66}, \\
\bar{S}_{ij}^{c(1)} &= \int_{-h_c/2}^{-h_c/6} Q_{ij}^{*c} z^2 dz, \quad \bar{S}_{ij}^{c(3)} = \int_{h_c/6}^{h_c/2} Q_{ij}^{*c} z^2 dz, \\
\bar{\Phi}_{1y}^{c(k^*)} &= \bar{H}_{11}^{c(k^*)} \bar{\alpha}_{11}^c + \bar{H}_{12}^{c(k^*)} \bar{\alpha}_{22}^c, \\
\bar{\Phi}_{1x}^{c(k^*)} &= \bar{H}_{12}^{c(k^*)} \bar{\alpha}_{11}^c + \bar{H}_{22}^{c(k^*)} \bar{\alpha}_{22}^c.
\end{aligned} \tag{7}$$

in which

- for trapezoidal corrugation

$$\begin{aligned}
c &= c_t, \quad l_c = c_t + 2f_t \left[\frac{1}{\sin(\alpha_t)} - \frac{1}{\tan(\alpha_t)} \right], \\
l_1 &= -\frac{8f_t \cos(\alpha_t)}{3 \sin(\alpha_t)} + 2c_t, \\
l_2 &= 2f_t^2 \left[c_t - \frac{2f_t}{\tan(\alpha_t)} \right] + \frac{4f_t^3}{3 \sin(\alpha_t)},
\end{aligned}$$

- for round corrugation

$$\begin{aligned}
c &= c_r = 2r_r, \quad l_c = \pi r_r + 2d_r, \quad l_1 = \pi r_r, \\
l_2 &= \pi r_r^3 + 8d_r r_r^2 + 2\pi d_r^2 r_r + \frac{4d_r^3}{3}.
\end{aligned}$$

and

$$\begin{aligned}
(H_{ij}, S_{ij}) &= \sum_{k=1}^{N_c} \int_{\Gamma^{(k)}} Q_{ij}^{(k)}(1, \zeta^2) d\zeta, \\
(\bar{\alpha}_{11}^c, \bar{\alpha}_{22}^c) &= \frac{1}{t_c} \sum_{k=1}^{N_c} \int_{\Gamma^{(k)}} [\alpha_{11}^{(k)}, \alpha_{22}^{(k)}] d\zeta, \\
Q_{ij}^{*c} &= \frac{12}{\left(\frac{h_c}{3}\right)^3} \bar{S}_{ij}^{c(2)},
\end{aligned} \tag{8}$$

where $\Gamma^{(k)}$ is the thickness domain of the k -th layer of the corrugation wall thickness, N_c is the total number of layers of the corrugation wall thickness. $\bar{S}_{ij}^{c(2)}$ are obtained by Eq. (6); for x-direction corrugation and for y-direction

corrugation, these can be observed by Eq. (7).

It can be seen that the coupling stiffness terms vanish due to the symmetric configuration of the sandwich shell with respect to the mid-surface. This symmetry leads to zero membrane-bending coupling terms, as the corresponding integrals of odd functions through the thickness cancel out.

3. Governing equation establishment

According to Donnell shell theory, the compatibility equation of the shell can be expressed as

$$\varepsilon_{x,yy}^0 + \varepsilon_{y,xx}^0 - \gamma_{xy,xy}^0 = w_{,xy}^2 - \frac{w_{,xx}}{R_y} - \frac{w_{,yy}}{R_x} \quad (9)$$

$$-w_{,xx}w_{,yy} - w_{,xx}w_{,yy}^* + 2w_{,xy}w_{,xy}^* - w_{,xx}^*w_{,yy}.$$

The stress function $\varphi(x, y)$ introduced in this study is required to satisfy the conditions

$$N_x = \varphi_{,yy}, \quad N_y = \varphi_{,xx}, \quad N_{xy} = -\varphi_{,xy}. \quad (10)$$

In this study, the panel configurations are examined under two types of boundary conditions.

First, all four edges are simply supported and freely movable (4F), defined as

$$w|_{x=0,a} = 0, N_{xy}|_{x=0,a} = 0, M_x|_{x=0,a} = 0, \\ N_x = N_{0x} = 0, \\ w|_{y=0,b} = 0, N_{xy}|_{y=0,b} = 0, M_y|_{y=0,b} = 0, \\ N_y = N_{0y} = 0, \quad (11)$$

Secondly, four simply supported edges are considered (2F2I), with two freely movable edges $x = 0$, $x = a$ and two immovable edges $y = 0$, $y = b$, as

$$M_x|_{x=0,a} = 0, N_x = N_{0x} = 0, N_{xy}|_{x=0,a} = 0, \\ w|_{x=0,a} = 0, \\ M_y|_{y=0,b} = 0, N_y = N_{0y}, v = 0, \\ w|_{y=0,b} = 0. \quad (12)$$

The deflection shape is approximated using a trigonometric function, and the initial imperfection is assumed to have an identical form, as presented in [37, 38]

$$w = W \sin(\alpha_m x) \sin(\beta_n y), \\ w^* = \xi h \sin(\alpha_m x) \sin(\beta_n y), \quad (13)$$

where m and n are the vibration modes,

$$\alpha_m = \frac{\pi}{a}m, \quad \beta_n = \frac{\pi}{b}n.$$

The compatibility equation in Eq. (9) is reformulated as

$$\bar{H}_{22}^* \varphi_{,xxxx} + (2\bar{H}_{12}^* + \bar{H}_{66}^*) \varphi_{,xxyy} + \bar{H}_{11}^* \varphi_{,yyyy} \\ + \frac{w_{,xx}}{R_y} + \frac{w_{,yy}}{R_x} - w_{,xy}^2 \\ + w_{,xx}w_{,yy} + w_{,xx}w_{,yy}^* - 2w_{,xy}w_{,xy}^* + w_{,xx}^*w_{,yy} = 0. \quad (14)$$

where

$$\bar{H}_{11}^* = \frac{\bar{H}_{22}}{\bar{H}_{11}\bar{H}_{22} - \bar{H}_{12}^2}, \quad \bar{H}_{12}^* = \frac{-\bar{H}_{12}}{\bar{H}_{11}\bar{H}_{22} - \bar{H}_{12}^2}, \\ \bar{H}_{22}^* = \frac{\bar{H}_{11}}{\bar{H}_{11}\bar{H}_{22} - \bar{H}_{12}^2}, \quad \bar{H}_{66}^* = \frac{1}{\bar{H}_{66}}.$$

By employing the harmonic balance method, the stress function can be derived upon inserting the assumed solution forms in Eq. (13) into Eq. (14), as

$$\varphi = \varphi_1 \cos 2\alpha_m x + \varphi_2 \cos 2\beta_n y \\ + \varphi_3 \sin \alpha_m x \sin \beta_n y + \frac{N_{0x}y^2}{2} + \frac{N_{0y}x^2}{2}. \quad (15)$$

where

$$\varphi_1 = \hat{V}_{11}W(W + 2\xi h), \\ \varphi_2 = \hat{V}_{21}W(W + 2\xi h), \\ \varphi_3 = \hat{V}_{31}W, \\ \hat{V}_{11} = \frac{\beta_n^2}{32\alpha_m^2\bar{H}_{22}^*}, \quad \hat{V}_{21} = \frac{\alpha_m^2}{32\beta_n^2\bar{H}_{11}^*}, \\ \hat{V}_{31} = \frac{\alpha_m^2/R_y + \beta_n^2/R_x}{\bar{H}_{11}^*\beta_n^4 + \alpha_m^2\beta_n^2(2\bar{H}_{12}^* + \bar{H}_{66}^*) + \bar{H}_{22}^*\alpha_m^4}.$$

In the average sense, the immovable edge condition for the two considered boundary cases is imposed as

$$\int_0^b \int_0^a v_{,x} dx dy \\ = C_{22}N_{0y} + C_{23}W(2\xi h + W) + C_{24}W + C_{25}\Delta T, \quad (16)$$

where

$$C_{22} = ab\bar{H}_{22}^*, \quad C_{23} = -\frac{ab\beta_n^2}{8},$$

$$\delta_x = (-1)^m - 1, \quad \delta_y = (-1)^n - 1,$$

$$C_{24} = -\frac{(\beta_n^2 \bar{H}_{12}^* + \alpha^2 \bar{H}_{22}^*) \hat{V}_{31} \delta_x \delta_y}{\alpha_m \beta_n} + \frac{\delta_x \delta_y}{\alpha_m \beta_n R_y},$$

$$C_{25} = ab(\bar{\Phi}_{1x} \bar{H}_{12}^* + \bar{\Phi}_{1y} \bar{H}_{22}^*).$$

The strain energy, the work of external loads, and the kinetic energy of the FG-GRMMC shells are respectively given by

$$\hat{U}_{int} = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^b \int_0^a \left[\sigma_x (\varepsilon_x - \alpha_{11} \Delta T) + \sigma_y (\varepsilon_y - \alpha_{22} \Delta T) + \sigma_{xy} \gamma_{xy} \right] dx dy dz, \quad (17)$$

$$\hat{U}_{ext} = \int_0^b \int_0^a q w dx dy, \quad (18)$$

$$\hat{U}_t = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^b \int_0^a \rho(z) (w_{,t} + w_{,t}^*)^2 dx dy dz. \quad (19)$$

To derive the equations of motion, the Lagrangian function is defined as

$$\hat{U} = \hat{U}_t - \hat{U}_{int} + \hat{U}_{ext}. \quad (20)$$

By employing the Rayleigh function of dissipation, the damping function is expressed as

$$d_v = \frac{1}{2} \int_0^b \int_0^a \tau w_{,t}^2 dx dy. \quad (21)$$

The equations of motion are derived by incorporating the Rayleigh dissipation function into the Euler-Lagrange formulation, expressed as

$$\frac{d}{dt} \left(\frac{\partial \hat{U}}{\partial \dot{W}} \right) - \frac{\partial \hat{U}}{\partial W} + \frac{\partial d_v}{\partial \dot{W}} = 0 \quad (22)$$

leads to

$$\begin{aligned} & C_{11} W + C_{12} W \left(W + \frac{4}{3} \xi h \right) \\ & + C_{13} W (2h\xi + W)(h\xi + W) + C_{15} N_{0y} \\ & + C_{18} q + C_{110} \tau \dot{W} + C_{111} \ddot{W} = 0 \end{aligned} \quad (23)$$

where

$$C_{11} = \frac{ab}{4} (\bar{H}_{22}^* \hat{V}_{31}^2 + \bar{S}_{11}) \alpha_m^4$$

$$+ \frac{ab}{4} \left[\hat{V}_{31}^2 (2\bar{H}_{12}^* + \bar{H}_{66}^*) + 2\bar{S}_{12} + 4\bar{S}_{66} \right] \beta_n^2 \alpha_m^2$$

$$+ \frac{ab}{4} (\bar{H}_{11}^* \hat{V}_{31}^2 + \bar{S}_{22}) \beta_n^4$$

$$C_{12} = -\frac{4\delta_x \delta_y \hat{V}_{31}}{\alpha_m \beta_n} \hat{V}_{11} \bar{H}_{22}^* \alpha_m^4$$

$$- \frac{4\delta_x \delta_y \hat{V}_{31}}{\alpha_m \beta_n} \left[\bar{H}_{12}^* (\hat{V}_{11} + \hat{V}_{21}) \alpha_m^2 \beta_n^2 + \hat{V}_{21} \bar{H}_{11}^* \beta_n^4 \right],$$

$$C_{13} = 16ab (\hat{V}_{11}^2 \alpha_m^4 \bar{H}_{22}^* + V_{21}^2 \beta_n^4 \bar{H}_{11}^*),$$

$$C_{15} = -\frac{\delta_x \delta_y \hat{V}_{31} (\bar{H}_{12}^* \beta_n^2 + \bar{H}_{22}^* \alpha_m^2)}{\alpha_m \beta_n}, \quad C_{18} = -\frac{\delta_x \delta_y}{\alpha_m \beta_n}.$$

Eq. (23) is applied for the 4F boundary condition combined with the movable edge $N_{0y} = 0$ and $N_{0x} = 0$. For 2F2I, the motion equation is applied by combining Eqs. (23) and (16). The equation of motion, written generally for the two considered boundary conditions, is presented as

$$\begin{aligned} & (\mu_1 C_{11} + \mu_2 J_{21}) W + C_{12} W \left(W + \frac{4}{3} \xi h \right) \\ & + C_{13} W (2h\xi + W)(h\xi + W) \\ & + \mu_2 J_{22} W (2h\xi + W) + \mu_2 J_{24} \Delta T \\ & + C_{18} q + C_{110} \tau \dot{W} + C_{111} \ddot{W} = 0 \end{aligned} \quad (24)$$

where $\mu_1 = 1, \mu_2 = 0$ for 4F, $\mu_1 = 0, \mu_2 = 1$ for 2F2I, and

$$J_{21} = C_{11} - \frac{C_{15} C_{24}}{C_{22}}, \quad J_{22} = -\frac{C_{15} C_{23}}{C_{22}}, \quad J_{24} = -\frac{C_{15} C_{25}}{C_{22}}.$$

For the forced vibration analysis, a time-dependent harmonic excitation $q = Q \sin \Omega t$ is introduced into Eq. (24), and the resulting nonlinear equation is solved using the Runge–Kutta method to obtain the time-deflection responses of the structure. In the case of free vibration, the nonlinear terms, damping effects, and external excitation are neglected, leading to a linear form of the governing equation. Consequently, the natural frequency can be determined as

$$\omega_{mn} = \sqrt{\frac{\mu_1 C_{11} + \mu_2 J_{21}}{C_{111}}} \quad (25)$$

Consider a perfect panel ($\xi = 0$), and introducing $q = Q \sin \Omega t$, Eq. (24) is re-expressed as

$$K_1 W + K_2 W^2 + K_3 W^3 + K_4 Q \sin \Omega t + K_5 \ddot{W} + K_6 \tau \dot{W} = 0 \quad (26)$$

where

$$K_1 = \mu_1 C_{11} + \mu_2 J_{21}, \quad K_2 = C_{12} + \mu_2 J_{22}, \\ K_3 = C_{13}, \quad K_4 = C_{18}, \\ K_5 = C_{111}, \quad K_6 = C_{110}.$$

The vibration amplitude is also assumed to be in a harmonic form

$$W = \Xi_1 \sin(\Omega t) + \Xi_2 \cos(\Omega t) \quad (27)$$

Substitution of the amplitude form given in Eq. (27) into Eq. (26), followed by the omission of higher-order terms and the application of the harmonic balance method, yields

$$-K_5 \Xi_1 \Omega^2 - K_6 \tau \Xi_2 \Omega + K_1 \Xi_1 + \frac{3}{4} K_3 \Xi_1^3 + \frac{3}{2} K_3 \Xi_1 \Xi_2^2 + K_4 Q = 0 \quad (28)$$

$$-K_5 \Xi_2 \Omega^2 + K_6 \tau \Xi_1 \Omega + K_1 \Xi_2 + \frac{3}{2} K_3 \Xi_1^2 \Xi_2 + \frac{3}{4} K_3 \Xi_2^3 = 0 \quad (29)$$

The polar coordinate system is applied to two amplitude components as

$$\Xi_1 = \Xi \sin(\Psi), \quad \Xi_2 = \Xi \cos(\Psi) \quad (30)$$

Substituting Eq. (30) into Eqs. (28) and (29), their approximated forms are obtained by

$$\Xi \sin(\Psi) \left(K_1 - K_5 \Omega^2 + \frac{15 K_3 \Xi^2}{16} \right) - \Xi \cos(\Psi) K_6 \tau \Omega = -K_4 Q \quad (31)$$

$$\Xi \cos(\Psi) \left(K_1 - K_5 \Omega^2 + \frac{15 K_3 \Xi^2}{16} \right) + \Xi \sin(\Psi) K_6 \tau \Omega = 0 \quad (32)$$

By solving Eqs. (31) and (32), the frequency-amplitude relation $\Omega - \Xi$ can be determined as follows

$$\Omega = \frac{1}{2} \sqrt{\pm \frac{\left(\frac{4K_1}{K_5} + \frac{15K_3\Xi^2}{4K_5} - \frac{2K_6^2\tau^2}{K_5^2} \right)}{\sqrt{\frac{4K_6^4\tau^4 - 16K_1K_5K_6^2\tau^2}{K_5^4} - \frac{15K_3K_6^2\tau^2\Xi^2}{K_5^3} + \frac{16K_4^2Q^2}{\Xi^2K_5^2}}}} \quad (33)$$

4. Numerical investigations and discussions

Table 1 presents a comparison of the fundamental natural frequencies of the present model with those reported in previous studies [10-12] for doubly curved panels without corrugated cores. A good agreement is observed between the present results and the reference solutions, indicating the accuracy and reliability of the proposed formulation.

Table 1. Comparison of the fundamental frequency parameters $\bar{\omega} = \omega_{mn} h \sqrt{\frac{\rho_c}{E_c}}$ of non-corrugated cores

Al_2O_3 double curved shallow shell panels (4F, $h/a = 0.1$, $a/b = 1$, $m = n = 1$, $\Delta T = 0K$).

a/R_x	b/R_y	References	
-0.5	0.5	Present	0.0581
		Matsunaga [10]	0.0563
		Chorfi and Houmat [11]	0.0580
		Alijani et al. [12]	0.0597
0.5	0.5	Present	0.0767
		Matsunaga [10]	0.0751
		Chorfi and Houmat [11]	0.0762
		Alijani et al. [12]	0.0779

Table 2. Comparisons of fundamental frequencies of doubly curved shell panels of the equivalent non-corrugated core and with layered corrugated core (rad/s, 4F, $h = 21\text{mm}$, $h_c = 15\text{mm}$, $a = b = 80h$, $R_x = R_y = a/0.1$, $T = 500\text{K}$, $m = n = 1$, $t = 1\text{mm}$, $f_t = 4\text{mm}$, $c_t = 4f_t$, $\alpha_t = \pi/4$, $r_r = 1.4\text{mm}$, $d_r = 0.6\text{mm}$, $c_r = 2r_r$)

	Trapezoidal corrugation			Round corrugation		
	UD	FG-X	FG-O	UD	FG-X	FG-O
Corrugated cores						
0/90/0	902.84	901.90	900.94	808.14	807.59	806.70
90/0/90	903.59	902.56	901.61	808.85	808.22	807.34
Equivalent non-corrugated cores						
0/90/0	747.59	747.33	746.60	767.76	767.41	766.68
90/0/90	747.63	747.36	746.63	767.80	767.44	766.71

Table 3. Effects of temperature and panel configuration on the fundamental natural frequency of doubly curved shell panels (rad/s, 2F2I, (90/90/0) shells, $h = 21\text{mm}$, $h_c = 15\text{mm}$, $a = b = 80h$, $m = n = 1$, $t = 1\text{mm}$, $f_t = 4\text{mm}$, $c_t = 4f_t$, $\alpha_t = \pi/4$, $r_r = 1.4\text{mm}$, $d_r = 0.6\text{mm}$, $c_r = 2r_r$)

T (K)	Trapezoidal corrugation			Round corrugation		
	UD	FG-X	FG-O	UD	FG-X	FG-O
$R_x = R_y = a/0.1$						
300	977.43	980.03	979.06	879.26	881.92	881.03
500	950.41	949.21	948.27	855.09	854.36	853.50
700	921.83	924.92	923.93	829.33	832.74	831.84
1000	873.89	871.81	870.91	788.05	786.80	785.95
$R_x = a/0.1, R_y = -a/0.1$						
300	637.35	638.95	637.70	575.27	576.71	575.47
500	619.36	618.36	617.17	559.01	558.14	556.97
700	600.80	602.46	601.20	542.25	543.76	542.51
1000	567.54	566.47	565.27	512.34	511.37	510.17

Table 2 compares the fundamental natural frequencies of sandwich panels with multilayer corrugated cores and those with equivalent non-corrugated cores, while maintaining the same amount of core material. It is observed that the corrugated configurations consistently exhibit higher natural frequencies than their non-corrugated counterparts, indicating a more efficient structural utilization of the same material. This enhancement is primarily attributed to the geometric arrangement of the corrugated core, which redistributes the material away from the mid-

surface and effectively increases the bending rigidity without increasing the total mass. In other words, the corrugated architecture improves stiffness through geometry rather than additional material, leading to superior dynamic performance. The influence of graphene distribution is also evident, with the UD pattern yielding slightly higher frequencies than FG-X and FG-O for the considered configurations. These results highlight that both corrugation geometry and material gradation are key factors in optimizing the vibration characteristics of sandwich structures, even under

the constraint of equal material usage.

Table 3 illustrates the effect of temperature on the fundamental natural frequencies of doubly curved sandwich panels with multilayer corrugated cores under 2F2I boundary conditions. It is observed that the natural frequencies decrease as the temperature increases. This reduction is not solely due to the temperature-dependent softening of material properties, but is also significantly influenced by the in-plane constraints at the panel boundaries. Under restrained boundary conditions, thermal expansion is partially prevented, leading to the development of in-plane compressive forces along the edges. These thermally induced membrane forces reduce the effective structural

stiffness and may introduce initial prebuckling deflections, both of which contribute to the decrease in natural frequencies. Therefore, the observed frequency reduction results from a combined effect of material degradation and geometric-mechanical coupling induced by thermal stresses. In addition, the influence of shell curvature remains evident. Panels with positive double curvature exhibit higher natural frequencies than saddle-shaped configurations due to their enhanced geometric stiffness. The graphene distribution also affects the response, although no single distribution pattern consistently provides the highest natural frequency across the investigated temperatures.

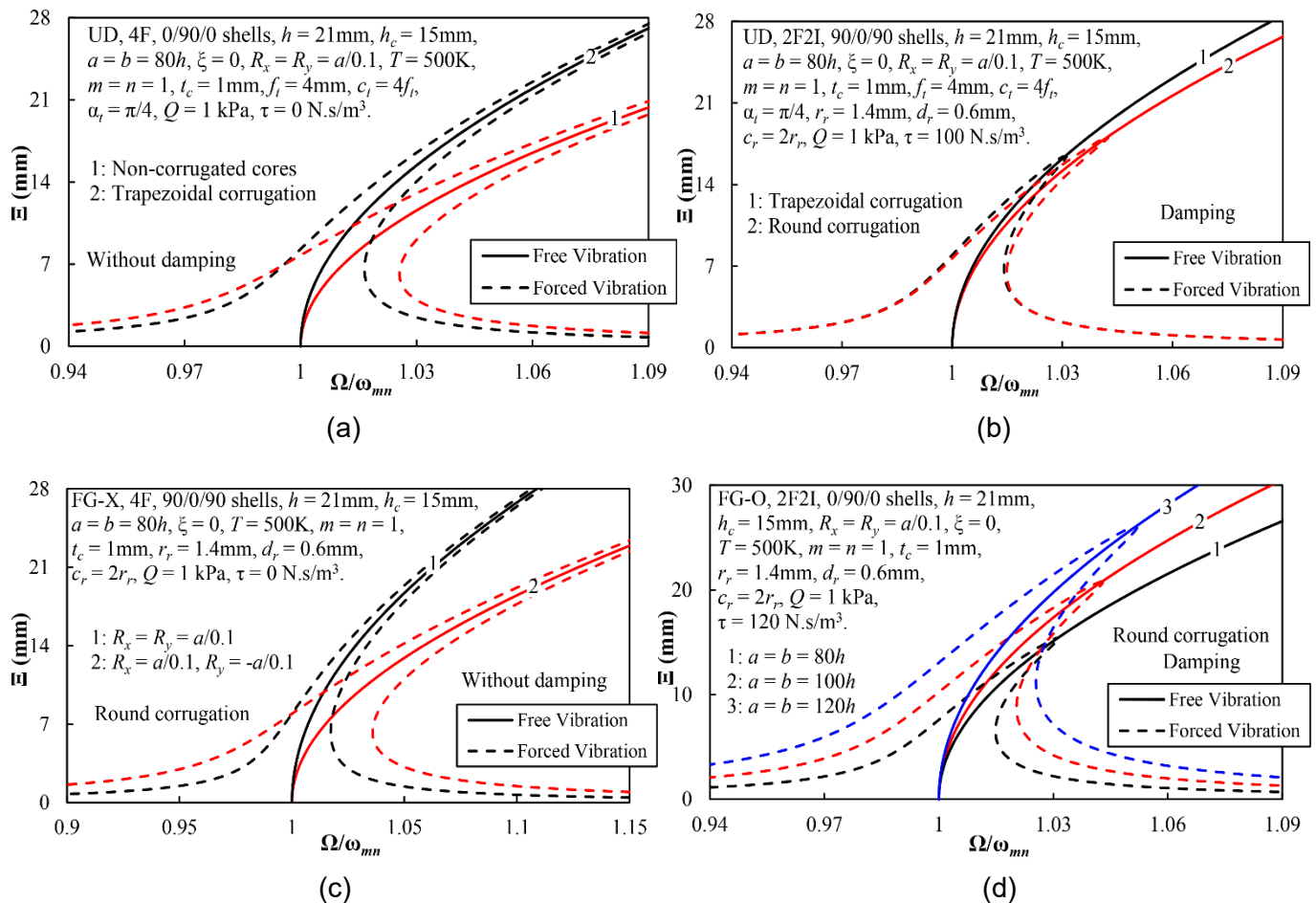


Fig. 3. Frequency-amplitude responses of doubly curved FG-GRMMC shell panels with multilayer corrugated FG-GRMMC core. (a) Comparison between non-corrugated core and trapezoidal corrugated core panels; (b) Comparison between trapezoidal corrugated core and round corrugated core panel; (c) Effects of curvature configurations; (d) Effects of edge lengths; (e) Effects of damping coefficients; (f) Effects of amplitudes of forced excitation

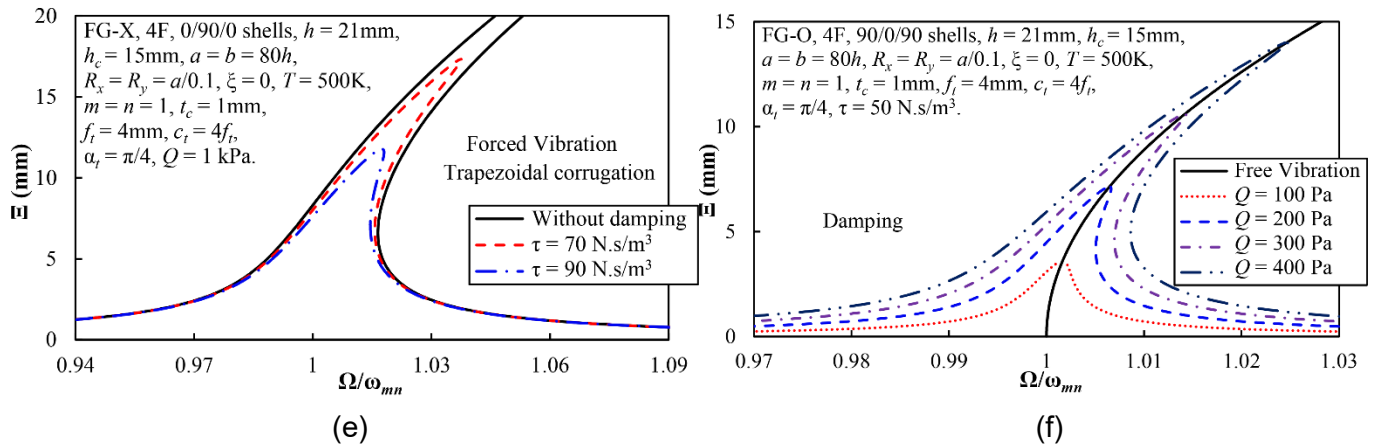


Fig. 3. (continued)

Fig. 3 presents the nonlinear frequency–amplitude responses of doubly curved FG-GRMMC sandwich panels with multilayer corrugated cores under different configurations. It can be clearly observed that all frequency–amplitude curves exhibit a typical nonlinear behavior, in which the natural frequency varies with vibration amplitude. In most cases, a hardening-type nonlinearity is observed, as the resonance curves bend toward higher frequencies with increasing amplitude, indicating the influence of geometric nonlinearity associated with large deflections. Fig. 3(a) shows that the panels with corrugated cores exhibit higher frequencies than those with equivalent non-corrugated cores over the entire amplitude range, confirming that the corrugated architecture enhances the effective stiffness not only in the linear regime but also in the nonlinear vibration domain. Fig. 3(b) further indicates that trapezoidal corrugation generally provides higher frequencies than round corrugation, due to its greater structural depth and stiffness contribution. The effect of shell curvature is illustrated in Fig. 3(c), where panels with positive double curvature demonstrate higher frequencies compared to saddle-shaped configurations, reflecting the additional geometric stiffness induced by curvature. In Fig. 3(d), increasing the panel dimensions leads to a reduction in frequency, which can be attributed to the decrease in global stiffness of the structure. The influence of damping

is presented in Fig. 3(e), showing that damping significantly reduces the vibration amplitude and smooths the nonlinear response, suppressing sharp resonance peaks. Finally, Fig. 3(f) indicates that increasing the excitation amplitude results in a pronounced nonlinear response, with larger vibration amplitudes and stronger deviation from linear behavior.

Fig. 4 illustrates the time-domain dynamic responses of doubly curved FG-GRMMC sandwich panels with multilayer corrugated cores under various geometrical configurations. It is observed that the structural response is strongly influenced by both the core architecture and the global geometric parameters of the panel. Fig. 4(a) shows that the incorporation of corrugated cores leads to a noticeable reduction in vibration amplitude compared to the non-corrugated configuration, indicating an enhancement in structural stiffness and dynamic stability. A similar trend is observed in Fig. 4(b), where trapezoidal corrugation generally produces smaller oscillation amplitudes than round corrugation, confirming its superior stiffness contribution. The effect of curvature is highlighted in Fig. 4(c), where panels with positive double curvature exhibit reduced vibration amplitudes compared to saddle-shaped configurations, due to the additional geometric stiffness provided by curvature. In Fig. 4(d), increasing the radii of curvature (i.e., reducing curvature) results in larger vibration amplitudes, reflecting the reduction in

structural rigidity. Fig. 4(e) demonstrates that increasing the panel dimensions leads to a significant amplification of vibration responses, which can be attributed to the decrease in global

stiffness and increased flexibility of the structure. Finally, Fig. 4(f) shows that the geometric parameters of the corrugation have a pronounced effect on the dynamic response.

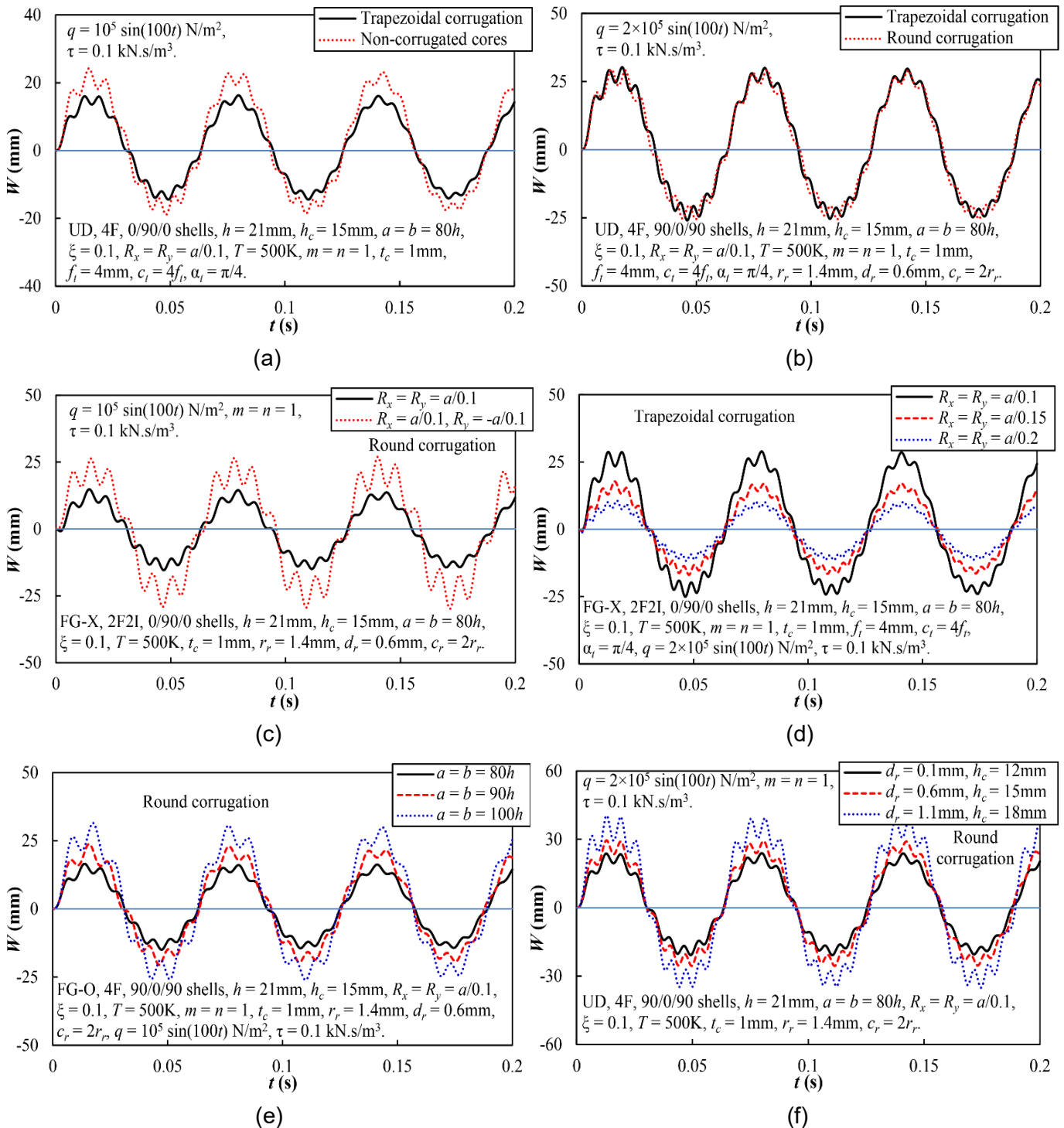


Fig. 4. Effects of geometrical characteristics on the dynamic behavior of doubly curved FG-GRMMC shell panels with multilayer corrugated FG-GRMMC core. (a) Comparison between non-corrugated core and trapezoidal corrugated core panels; (b) Comparison between trapezoidal corrugated core and round corrugated core panel; (c) Effects of curvature configurations; (d) Effects of radii of panels; (e) Effects of edge lengths; (f) Effects of geometrical properties of round corrugation

5. Conclusion

This paper has presented an analytical study on the nonlinear vibration behavior of doubly curved sandwich panels with multilayer corrugated cores made of FG-GRMMC. An improved homogenization technique was employed to evaluate the effective properties of the layered corrugated core, and the Ritz method was applied, while the governing equations were formulated based on nonlinear large-deflection Donnell shell theory.

The results show that the corrugated core configuration significantly enhances the dynamic performance of the sandwich panels compared to equivalent non-corrugated cores, even under the same material usage. The nonlinear frequency-amplitude responses reveal a typical hardening behavior, indicating the strong influence of geometric nonlinearity.

It is also found that temperature has a pronounced effect on the vibration characteristics, leading to a reduction in natural frequencies due to both material softening and thermally induced in-plane compressive forces under restrained boundary conditions. In addition, the shell curvature plays an important role, with positively curved panels exhibiting higher stiffness and lower vibration amplitudes compared to saddle-shaped configurations. The distribution of graphene further influences the vibration characteristics, with its effects depending on temperature and the considered structural configuration.

The findings of this study provide useful insights for the design and optimization of advanced sandwich shell structures with complex core architectures under dynamic loading environments.

Declaration of Generative AI

During the preparation of this work, the authors did not use Generative AI and AI-assisted technology tools in order to generate the content of the manuscript.

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